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HIGH-SPEED COMPRESSIBLE FLOWS ABOUT AXISYMMETRIC BODIES

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ABSTRACT

The approximate method for the account of compressibility, which was successfully applied earlier for 2-D problems, is developed for axisymmetric case in the present work. The offered method is analyzed with reference to two mediums: air and water. Comparison of the results of calculations on an offered method and a method of Dorodnitsyn integrate relations has shown acceptable accuracy of the proposed method for engineering calculations. The method allows determining parameters of compressible flow through the parameters of incompressible flow up to the velocity corresponding to critical Mach number. The method of the account of compressibility of a flow does not depend on mathematical model of calculation of incompressible flow.

Calculations have shown that for the same body the value of critical Mach number in water is less, than in air. The effect of compressibility is shown in water more strongly and at smaller values of Mach numbers of the flow infinity, than in air.

INTRODUCTION

Modern development of hydrodynamics is characterized by achievement big subsonic and even transonic and supersonic speeds of movement of bodies in water [1, 2, 3]. Such big speeds until recently were inherent only for the bodies moving in air. The higher the Mach number of the free stream and the level of disturbances produced by the body at low Mach numbers, the stronger the effects of compressibility of the flow. The compressibility at movement of bodies becomes actual for such medium as water, which earlier traditionally was considered incompressible. As it is known, movement of bodies in water may occur both at advanced cavitation mode (supercavitation), and at a mode of continuous flow. The knowledge of critical value of Mach number at which somewhere on a surface of a body local speed of flow reaches local speed of a sound, will allow making a correct choice of the governing equations of hydrodynamics for movement of a body with the high speed.

As it is known, at the transition of speed of sound the equations change one's own type from elliptic at subsonic speeds to hyperbolic at supersonic. Value of critical Mach number may serve as the upper boundary of application of the equations of elliptic type.

The approximate method for the account of compressibility, which was successfully applied earlier for 2-D problems [5] is developed for axisymmetric case in the present work. The offered method is analyzed with reference to two mediums: air and water.

Methods of the account of compressibility in two-dimensional flows enough well are developed [6-10]. The wide application was obtained by a method based on a model of Chaplygin gas [10]. The drawback of this method is the necessity to search for velocity potential and stream function of a flow, which should satisfy the linear equations of gas dynamics in a plane of a hodograph of velocity. These equations have received the name of Chaplygin equations. Usually the theory of function of complex variable is applied to the solution of Chaplygin equations. The solution of these equations in a case of adiabatic flows is connected to the big difficulties, as it is not always possible to formulate boundary conditions in a plane of hodograph of velocity. However, in case of barotropic gas model it is possible to consider the approximate model, which has received the name "Chaplygin gas model". The solution of a problem for Chaplygin gas model is carried out by method of successive approximations. For zero approximation, the functions satisfying an incompressible liquid are accepted. Chaplygin method has obtained development in S. A. Khristianovich's works [11].

Analytical solution to the problem of a compressible gas flow around a single circle was obtained by A. I. Nekrasov [12] based on Legendre transformations method. For an elliptic contour, such problem with Legendre transformations method was considered by L. K. Kudriashov [13] and with a method of decomposition of velocity potential in a series in terms of powers of M_∞^2 – by C. Kaplan [14]. S. A. Khristianovich, I. M. Yuriev [15] and L. I. Sedov [7] obtained solutions for ellipses by methods of hodograph of velocity. The drawback of these methods is that there is a deformation of a streamline contour, so the obtained solutions describe flows around some ovals instead of initial ellipses. The method of approximation of an adiabatic for the solution of a problem of a flow around a circle and an ellipse has been developed and applied by G. A. Dombrovski [9]. Numerical solutions based on a method of Dorodnitsyn integrate relations with the account of compressibility for two-dimensional flows around single circular and elliptic contours, and airfoils were obtained in works of P. I. Chushkin [16, 17], R. Melnic and D. C. Ives [18], M. Holt and B. Meson [19].

The review of the methods which are taking into account compressibility in two-dimensional flows is contained in works of H. W. Liepmann and A. E. Puckett [6], Shih-I Pai [8], G. A. Dombrovski [9] and L. I. Sedov [7], but basically these methods were directed on solutions of problems of compressible gas flow around the thin bodies such as airfoils. If the body is bluff, the disturbances from it at the high Mach numbers will be significant, therefore all methods using the assumption about a little relative thickness can't be applied. Chaplygin method is based on application of a theory of complex variable function, which remains the tool for the solution of two-dimensional problems only.

The problem of axisymmetric compressible flows of liquid around spheroids was considered. These bodies were chosen because there are exact analytical solutions to a velocity potential of incompressible flow around such bodies [5, 29]. Many results were got for a sphere. Kaplan [14] has obtained for a sphere an approximation with accuracy M_∞^4 that has allowed him to define the value of critical Mach number $M_* = 0{,}573$. Schmieden (1942) (see work [16]), using Rayleigh-Janzen method, has calculated the approximate solution only for thick spheroids $(\delta \geq 0{,}8)$. Hida [21] based on Poggi method with approximation with accuracy M_∞^2 has obtained the solution also only for thick spheroids. For axisymmetric case of compressible flow Chushkin [16, 17] has applied a method of Dorodnitsyn integrate relations using elliptic coordinates, for ellipsoids of revolution.

In the listed works as medium air was considered. For water, the account of compressibility for cavitation flows was considered by Vasin [22, 23], Zigangareeva and Kiselev [24, 25], Serebryakov [1], and for a gliding – by Mayboroda [30].

In the present work the approximate method for the account of compressibility, which can be applied both for cavitation flows, and for continuous flow, is offered. It is based on the potential model of the flow, which is frequently used for problems of cavitation and continuous flow. The proposed method for the account of compressibility of a flow is based on Burago method [26], which was developed and applied originally by author for airfoils. Burago method was described also in monograph by Arjanikov and Maltsev [27]. This method does not impose any restrictions on thickness of a body, it is fast and, as it have been shown by calculations for two-dimensional problems [4], it is nearly as accurate as Khristianovich method [11, 15].

In the present work, applicability of Burago method for the account of compressibility of flows around the axisymmetric bodies moving both in air and in water is analyzed. As the method is approximate, much attention we must give the matter approbation of this method. Problems of continuous flow around ellipses (two-dimensional case) and ellipsoids of revolution (axisymmetric case) are chosen as test cases.

NOMENCLATURE

a = speed of sound [see formula (16)];
B = constant value in Tait's equation of state of the water [see formula (6)];
c_p $= \dfrac{p - p_\infty}{\rho_\infty U_\infty^2 / 2}$ – pressure-drop coefficient;
h = enthalpy;
M = Mach number;
M_* = critical Mach number;
n = index of a problem $(n = 0$ or $n = 1)$;
p = pressure;
$P(p)$ = pressure function [see formula (7)]
$\bar{u}, \bar{v}$ = $\bar{u} = u/U_\infty$ and $\bar{v} = v/U_\infty$ – dimensionless velocities;
U_∞ = stream velocity;
V = vector of full local speed of flow;
x, r = cylindrical coordinates;
δ = relative thickness of an ellipse or ellipsoid;
ε = error of calculations;
η_c = compressibility factor of the flow [see formula (13)];
κ = ratio of specific heats (for air $\kappa = 1{,}4$; for water $\kappa = 7{,}15$);
ρ = local density of medium;
σ, τ = special gas-dynamic function [see formula (3)];

Subscripts

∞ = lower index, specifies parameters of the flow at infinity;;
0 = lower index, specifies parameters in stagnation point; upper index, specifies parameters for incompressible flow;
* = lower index, specifies critical parameters.

THE APPROXIMATE MODEL OF THE COMPRESSIBILITY OF THE FLOW

Let's write down the equations of continuity and existence of velocity potential for two-dimensional and axisymmetric incompressible irrotational flow of liquid

$$\frac{\partial (r^n \rho u)}{\partial x} + \frac{\partial (r^n \rho v)}{\partial r} = 0\,, \qquad (1)$$

$$\frac{\partial u}{\partial r} - \frac{\partial v}{\partial x} = 0\,, \qquad (2)$$

where x, r – cylindrical coordinates; u, v – component of velocity along axis x и r accordingly;

$$n = \begin{cases} 0 & - \quad 2-D \text{ case} \\ 1 & - \quad \text{axisymmetric case} \end{cases}.$$

For axisymmetric case, coordinates (x, r) are entered in meridian plane of flow.

Introduction of the special functions offered Burago [26] as

$$\tau = \frac{2}{\dfrac{\rho_0}{\rho} + 1}; \quad \sigma = \frac{\dfrac{\rho_0}{\rho} - 1}{\dfrac{\rho_0}{\rho} + 1} \qquad (3)$$

allows the equation of continuity and condition of existence of velocity potential of a flow to be written as

$$\begin{aligned} &\frac{\partial}{\partial x}\left(\frac{\tau\, u\, r^n}{1+\sigma}\right) + \frac{\partial}{\partial r}\left(\frac{\tau\, v\, r^n}{1+\sigma}\right) = 0, \\ &\frac{\partial}{\partial r}\left(\frac{\tau\, u}{1-\sigma}\right) - \frac{\partial}{\partial x}\left(\frac{\tau\, v}{1-\sigma}\right) = 0. \end{aligned} \qquad (4)$$

For barotropic model of compressible liquid we have:

for air [5]

$$\frac{p}{p_\infty} = \left(\frac{\rho}{\rho_\infty}\right)^\kappa, \kappa = 1{,}40 \qquad (5)$$

for water [28]

$$p - p_\infty = B\left[\left(\frac{\rho}{\rho_\infty}\right)^\kappa - 1\right], \kappa = 7{,}15\,, \qquad (6)$$

$$B = \frac{\rho_\infty a_\infty^2}{\kappa} \approx 298 \;\; MPa, a_\infty = 1460\; m/s, \rho_\infty = 1000\; kg/m^3$$

The equation (5) refers to as Poisson adiabatic curve, and the equation (6) – Tait formula. It is considered that pressure in water does not exceed 2940 *MPa*. Thus, static and dynamic adiabatic curves for water practically coincide and are described by Tait equation [6].

By virtue of accepted barotropic model of liquid the values of enthalpy and pressure function differ only on a constant [5]

$$h = \mathrm{P}(p) + const\,,$$

where pressure function is determined by the formula

$$\mathrm{P}(p) = \int_{p_\infty}^{p} \frac{dp}{\rho} \qquad (7)$$

Substitution of Tait formula in (7) and integration gives the formula for enthalpy

$$h = \frac{\rho^{\kappa-1}}{(\kappa-1)\rho_\infty^{\kappa-1} M_\infty^2} \qquad (8)$$

Neglecting action of mass forces, and using the equation for enthalpy (8), it is possible to present the Bernoulli equation for compressible liquid as

$$\frac{\bar{V}^2}{2} + \frac{\rho^{\kappa-1}}{(\kappa-1)\rho_\infty^{\kappa-1} M_\infty^2} = \frac{1}{2} + \frac{1}{(\kappa-1) M_\infty^2}\,, \qquad (9)$$

From (9) it is possible to get relationships between density and speed of sound and velocity of the flow

$$\begin{aligned} \frac{\rho}{\rho_\infty} &= \left[1 + \frac{\kappa-1}{2} M_\infty^2 \left(1 - \bar{u}^2 - \bar{v}^2\right)\right]^{\frac{1}{\kappa-1}}, \\ \frac{a}{a_\infty} &= \left[1 + \frac{\kappa-1}{2} M_\infty^2 \left(1 - \bar{u}^2 - \bar{v}^2\right)\right]^{\frac{1}{2}}, \end{aligned} \qquad (10)$$

Using stagnation flow parameters, it is possible to write down known isentropic relations for density and speed of sound [5]

$$\begin{aligned} \frac{\rho_0}{\rho} &= \left(1 + \frac{\kappa-1}{2} M^2\right)^{\frac{1}{\kappa-1}}, \\ \frac{a_0}{a} &= \left(1 + \frac{\kappa-1}{2} M^2\right)^{\frac{1}{2}}. \end{aligned} \qquad (11)$$

Formulas (10), (11) are valid both for air and for water. Thus, speed of sound determined for any medium as

$$a = \sqrt{\frac{dp}{d\rho}}$$

for air in the case adiabatic process it is equal [5]

$$a = \sqrt{\kappa \frac{p}{\rho}}\,, \qquad (12)$$

and for water, using the state equation in the form (6), it is possible to get

$$a = a_\infty \left(\frac{\rho}{\rho_\infty}\right)^{\frac{\kappa-1}{2}} \qquad (13)$$

Using isentropic relations for density (10) is possible to calculate function σ (3). In Fig.1, plot of function σ for air and water versus Mach number is shown in limits from zero up to 1,0.

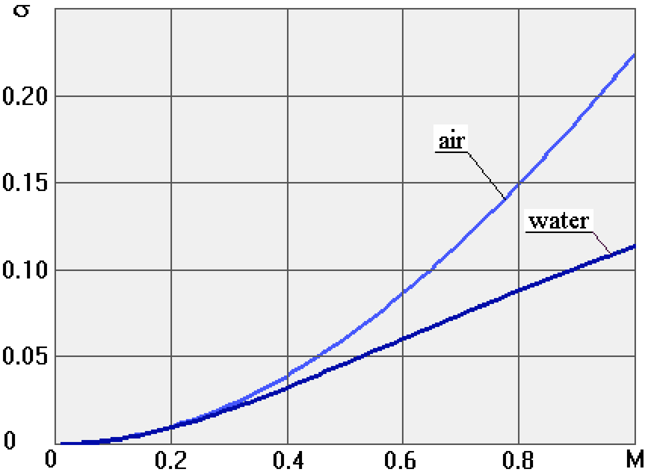


Fig.1: Dependence of function σ from Mach number

From Fig.1 it can be seen, that values of function σ lay in a range [0; 0,22] for air and [0; 0,11] for water for local Mach number M from 0 up 1,0. This circumstance allows considering $\sigma \approx const$, and based on formulas (3), equations (4) can be replaced by the approximated ones

$$\begin{aligned} &\frac{\partial}{\partial x}\left(\frac{\rho\, u\, r^n}{\rho_0 + \rho}\right) + \frac{\partial}{\partial r}\left(\frac{\rho\, v\, r^n}{\rho_0 + \rho}\right) = 0, \\ &\frac{\partial}{\partial y}\left(\frac{\rho\, u}{\rho_0 + \rho}\right) - \frac{\partial}{\partial r}\left(\frac{\rho\, v}{\rho_0 + \rho}\right) = 0. \end{aligned} \qquad (14)$$

Let's introduce new variables u^0 and v^0, connected to u and v by formulas

$$u = \eta_c u^0; \quad v = \eta_c v^0\,, \qquad (15)$$

where the parameter is introduced

$$\eta_c = \frac{1 + \dfrac{\rho_0}{\rho}}{1 + \dfrac{\rho_0}{\rho_\infty}}\,, \qquad (16)$$

which will be called “the compressibility factor of the flow”.

Using the new variables (15), equations (154) will lead to

$$\begin{aligned} &\frac{\partial (r^n u^0)}{\partial x} + \frac{\partial \left(r^n v^0\right)}{\partial r} = 0, \\ &\frac{\partial u^0}{\partial r} - \frac{\partial v^0}{\partial x} = 0. \end{aligned} \qquad (17)$$

Equations (17) are the well-known equations describing the two-dimensional $(n = 0)$ or axisymmetric $(n = 1)$ potential flow of an incompressible liquid (see equation (1), (2) for $\rho = const$). The velocity $\mathbf{V}^0$ of this flow has components (u^0, v^0). The formulas (15) allow us to prove that the boundary conditions at infinity (far from the body) for the flow of compressible gas with components of velocity (u, v), will be identical with the appropriate boundary conditions for an incompressible flow with components of velocity (u^0, v^0). Boundary conditions on a surface of the body will be identical also. Really, on a surface of the body in the compressible flow $v_n = 0$. Based on the equations (15), (16) on the surface of the body in an incompressible flow we have

$$v_n^0 = \frac{\rho}{\rho_\infty} \frac{(\rho_\infty + \rho_0)}{(\rho + \rho_0)} v_n = 0 .$$

Thus, the equations (15) gives the approximated relationship between velocities in compressible and incompressible flow around the same body under identical conditions at infinity and on a surface of the body. Let's write the differential equation for streamlines of two-dimensional or axisymmetric flow [5]

$$\frac{dx}{u} = \frac{dr}{v} \tag{18}$$

The substitution in the equation (18) of the compressible flow velocities, expressed via compressibility factor of the flow (15), and the velocity of an incompressible flow, enables us to prove, that the form of streamlines in compressible and incompressible flows will be identical. This circumstance is identified as the hypothesis of "stabilization of the streamlines". To calculate velocities of compressible gas flow, it is necessary to use the well-known isentropic formulas for the relations of density and relation of local speed of a flow to the speed of sound at the stagnation point of a flow

$$\begin{aligned} &\frac{\rho_0}{\rho_\infty} = (1 + \frac{\kappa - 1}{2} M_\infty^2)^{\frac{1}{\kappa - 1}}, \\ &\frac{|\mathbf{V}|}{a_0} = \frac{M}{\sqrt{(1 + \frac{\kappa - 1}{2} M^2)}} \end{aligned} . \tag{19}$$

In the last formula (19) the left part can be written as

$$\frac{|\mathbf{V}|}{a_0} = \frac{U_\infty \, a_\infty \sqrt{\bar{u}^2 + \bar{v}^2}}{a_\infty \, a_0} = \frac{M_\infty \sqrt{\bar{u}^2 + \bar{v}^2}}{\sqrt{(1 + \frac{\kappa - 1}{2} M_\infty^2)}} . \tag{20}$$

From (19) and (20) it follows

$$M = M_\infty \sqrt{\frac{\bar{u}^2 + \bar{v}^2}{1 + \frac{\kappa - 1}{2} M_\infty^2 (1 - \bar{u}^2 - \bar{v}^2)}} . \tag{21}$$

As can be seen from formula (21), the local Mach number is defined by local dimensionless velocities $\bar{u}$ и $\bar{v}$, which, according to the formulas (15), (16) and isentropic formulas for density depend on local Mach number M. To take this into account, it is necessary to use successive approximations. At the first stage of approximation, the values of velocities calculated for incompressible flow are substituted in (21)

$$\bar{u}^{(1)} = \frac{u^0}{U_\infty}; \quad \bar{v}^{(1)} = \frac{v^0}{U_\infty} . \tag{22}$$

Based on these values, the local Mach number $M^{(1)}$ for the first approximation is defined. The obtained value of Mach number $M^{(1)}$ is used for calculation of local velocities of a compressible flow at the second approximation stage $\bar{u}^{(2)}$ и $\bar{v}^{(2)}$ from the formula (15), (16) and isentropic formulas for density of liquid (19). The obtained values of velocities allow us to calculate, based on the formula (21), the local Mach number for the second approximation stage $M^{(2)}$, which is in turn used for calculation of local velocities at the third approximation stage $\bar{u}^{(3)}$, $\bar{v}^{(3)}$ and so on. The approximation process needs to be continued until the following inequality will be satisfied

$$\left|\eta_c^{(n)} - \eta_c^{(n-1)}\right| - \left|\eta_c^{(n-1)} - \eta_c^{(n-2)}\right| \le \varepsilon .$$

The calculations based on the described algorithm have shown, that the number of approximation stages increases while $M \to 1{,}0$, but $n_{\max} < 30$.

Formulas (19) and (20) yield the equality

$$\frac{M_\infty \sqrt{\bar{u}^2 + \bar{v}^2}}{\sqrt{1 + \frac{\kappa - 1}{2} M_\infty^2}} = \frac{M}{\sqrt{1 + \frac{\kappa - 1}{2} M^2}},$$

from which, considering $M = 1$ and $M_\infty = M_*$, it is possible to obtain the formula for calculation of critical Mach number as

$$M_* = \sqrt{\frac{1}{\bar{u}^2 + \bar{v}^2 + \frac{\kappa - 1}{2} (\bar{u}^2 + \bar{v}^2 - 1)}} \tag{23}$$

or

$$M_* = \sqrt{\frac{1}{\bar{U}^2 + \frac{\kappa - 1}{2} (\bar{U}^2 - 1)}} , \tag{24}$$

where the designation for local total relative velocity $\bar{U}$ is introduced. From formula (24) it follows, that the minimal value of M_* corresponds to the maximal value $\bar{U}$. To define critical Mach number for flow around a contour, it is necessary to find the maximum local velocity in the flow around this contour. In formula (23) the dimensionless velocities $\bar{u}$ and $\bar{v}$ also should correspond to the critical Mach number of forward flow; therefore, for calculation of critical Mach number, we apply a method of successive approximations in the same way as it was done for calculation of local Mach number upon the formula (21). As the first approximation, we shall substitute in formula (23) the values of relative local velocities $\bar{u}^{(1)}$ и $\bar{v}^{(1)}$, calculated for an incompressible flow. The critical Mach number $M_*^{(1)}$, obtained at the first approximation stage, is to be considered as the Mach number of the free flow for calculation of local relative velocities for the second approximation stage $\bar{u}^{(2)}$ и $\bar{v}^{(2)}$ from the formulas (15). The process of the approximation needs to be continued until the calculated critical Mach number for subsequent approximations will differ on the given error ε, i.e., until condition

$$\left|M_*^{(i)} - M_*^{(i-1)}\right| \le \varepsilon$$

will be satisfied.

As can be seen, the critical Mach number can be calculated using only the values of the velocities of an incompressible flow. It is necessary to notice that the method described above is applicable for compressible flows, for which the inequality $M_\infty < M_*$ is valid. This condition specifies the absence of transonic and supersonic zones in a field of flow. As it can be seen from the method described above, for calculation of critical Mach number M_* and velocity field (u, v) in compressible flow, it is enough to calculate the velocity field (u^0, v^0) about a body for an incompressible flow.

Based on the isentropic formulas (10) и (19) it is possible to express the pressure-drop coefficient for compressible flow through local Mach number and Mach number of the flow at infinity

$$c_p = \frac{2}{\kappa M_\infty^2}\left[\left(\frac{1+\frac{\kappa-1}{2}M_\infty^2}{1+\frac{\kappa-1}{2}M^2}\right)^{\frac{\kappa}{\kappa-1}} - 1\right]. \qquad (25)$$

Note that the formula (25) for water can also be obtained directly from a static adiabat (6). Local Mach number in the formula (25) is calculated by the method of successive approximations based on the formula (21) as it was described above.

RESULTS OF CALCULATIONS

Since the proposed method for the account of compressibility is approximate, it is expedient to compare it with other methods to analyze accuracy of calculation. In work [4] there were given some a number of comparisons for 2-D problems and air only. Comparisons have shown that the accuracy of calculations on proposed method is high enough.

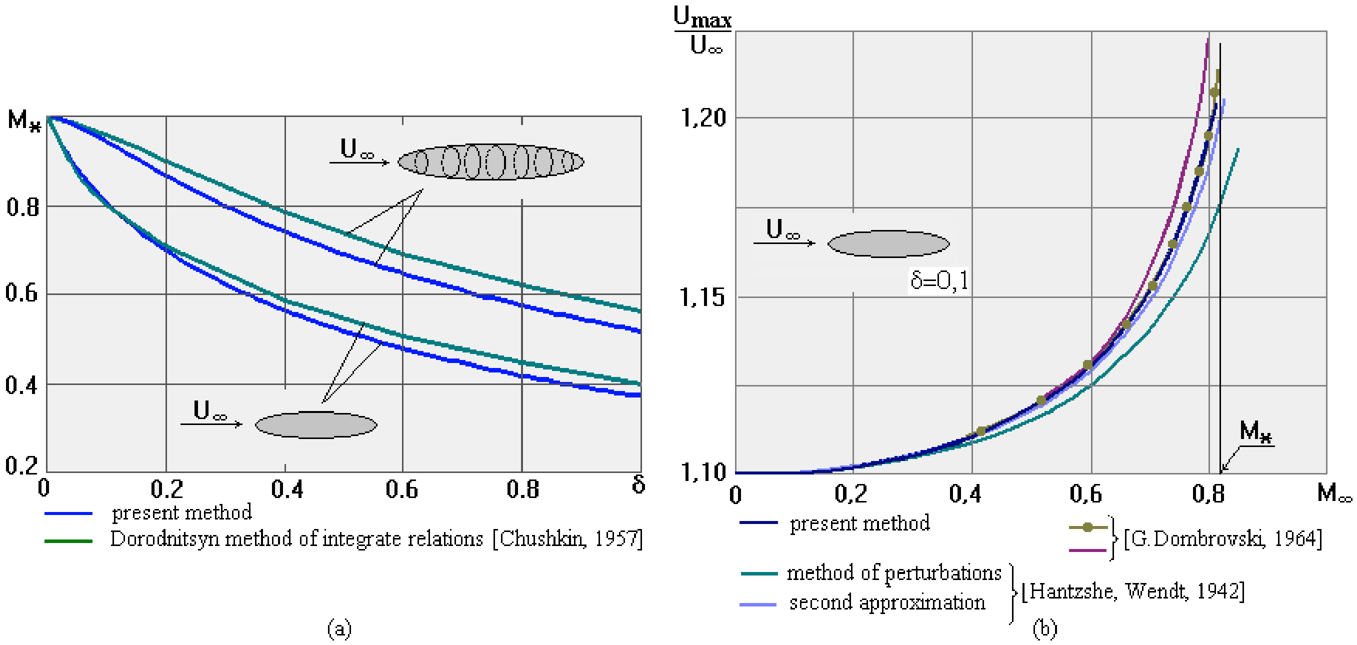
Additional calculations for two-dimensional problems are analyzed here and calculations for axisymmetric flow are executed. Ellipses and ellipsoids of revolution (spheroids) with various factor of compression δ are chosen. For calculations, the well-known potential models for ellipses and spheroids [5, 29] were used. It is obvious, that the maximal error will correspond to flow around thick bodies $(\delta \to 1{,}0)$ and for case $M_\infty = M_*$. In Table 1 and in Fig.2(a) results of calculation of critical Mach number for ellipses and spheroids with an offered method and with method of integrate relations, executed by Chushkin [16], are compared.

The comparison in Table 1 and Fig.2(a) shows a good agreement. It is necessary to notice, that calculations in a range of $0{,}4 \le \delta \le 0{,}8$ were executed by Chushkin only for the second approximation, and each subsequent approximation resulted in reduction of critical Mach number. Therefore, some additional error can be explained by this fact. As a whole, it is possible to note, that the relative difference increases with the increase of value δ, both for ellipses and for spheroids. However, the maximal relative difference does not exceed 7 % for a circle, and 8 % - for a sphere.

Fig.2(b) shows comparison of the maximal velocity on an elliptic contour to velocity on infinity ratio versus Mach number (symmetric flow). Relative thickness of an ellipse is $\delta = 0{,}1$. Here alongside with calculations on an proposed method the calculation results obtained from the theory of small disturbances and from second approximation by Hantzshe and Wendt (1942, see work [9]), and the calculation results obtained on a method of approximation of adiabat by Dombrovski [9] are shown. The calculation results obtained on a proposed method have practically coincided with results of Dombrovski. The critical Mach number $M_* = 0{,}807$ is calculated on an approximate method. It agrees very well with the data of Lighthill (1954, see work [9]) and Dombrovski [9] ($M_* \approx 0{,}81$) and Chushkin $M_* = 0{,}803$ (Table 1). Earlier results, which were obtained by Kaplan [14] $M_* \approx 0{,}857$ should be considered less exact though the relative difference for this result does not exceed 6,2 %.

Table 1: Comparison of values for critical Mach number at a flow of ellipses and spheroids

δ	Ellipses				Ellipsoids			
	$C^0_{p\,\min}$	Chushkin [16] M_*	Calculation M_*	Relative difference, %	$C^0_{p\,\min}$	Chushkin [16] M_*	Calculation M_*	Relative difference, %
0,05	-0,1025	0,869	0,884	1,77	-0,0136	0,984	0,980	0,41
0,1	-0,2100	0,803	0,807	0,50	-0,0418	0,957	0,945	1,25
0,15	-0,3225	0,752	0,748	0,53	-0,0801	0,929	0,905	2,58
0,20	-0,4400	0,709	0,700	1,27	-0,1219	0,899	0,868	4,00
0,40	-0,9600	0,588	0,566	3,74	-0,3370	0,783	0,742	5,24
0,60	-1,5600	0,506	0,480	5,14	-0,6022	0,692	0,648	6,36
0,80	-2,2400	0,447	0,418	6,49	-0,9078	0,620	0,576	7,10
1,00	-3,0000	0,399	0,372	6,77	-1,2500	0,563	0,519	7,82



(a) (b)

Fig.2: a) Comparison of critical Mach number versus relative thickness of an ellipse and ellipsoid;

b) Comparison of relative maximal velocity in a flow about ellipse versus Mach number

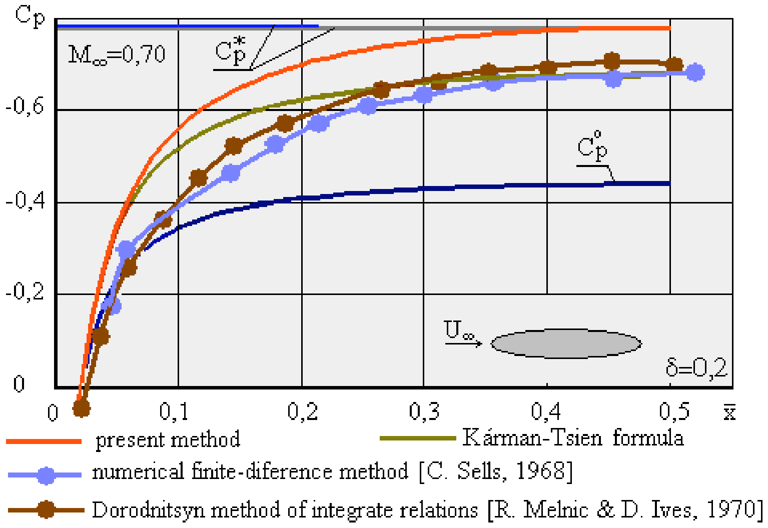
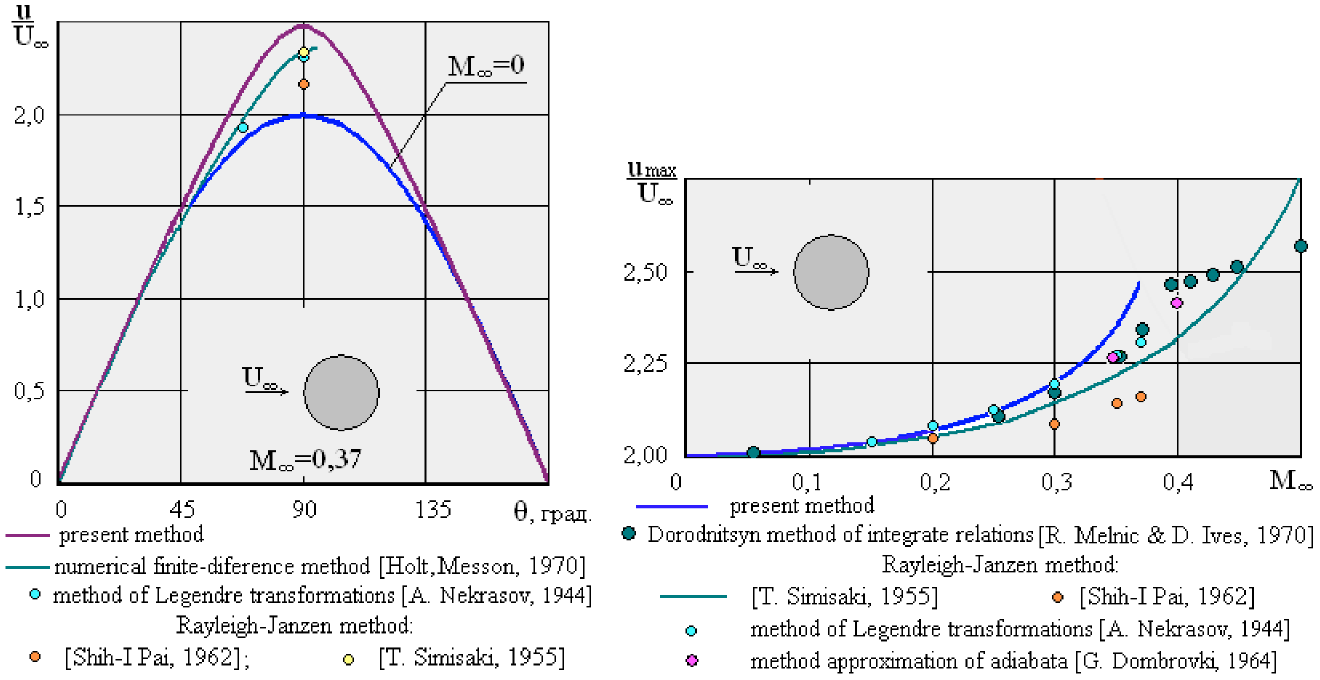


Fig.3 Comparison of the pressure-drop coefficient for an ellipse with relative thickness of equal 20%

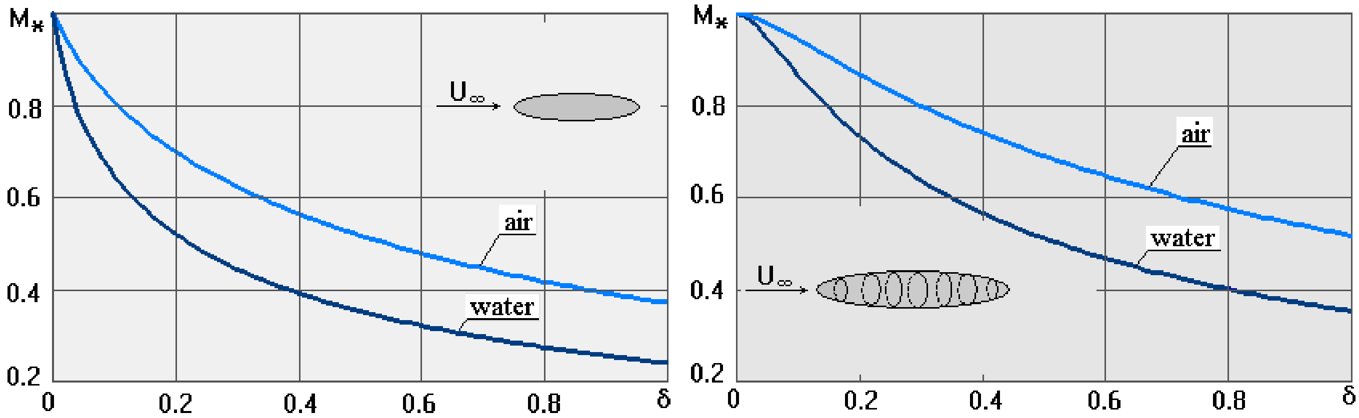
Fig.3 shows comparison of the calculation results on pressure-drop for an ellipse of relative thickness $\delta = 0,2$ in the compressible flow. Calculations were carried out for Mach number of an infinity flow $M_\infty = 0,70$, which is critical Mach number for an ellipse with such relative thickness (Table 1). The calculated data on a method of finite-differences Sells (1968) are taken from work [18]. Approximately up to values $\bar{x} \le 0,1$ the results of Burago method and of Kárman-Tsien formula [5, 7] are in good agreement. For values of relative coordinate along the big axis of an ellipse $0,1 < \bar{x} \le 0,5$ the significant divergence between the results of the Kárman-Tsien formula and Burago method is observed. Despite that the data of other authors are in better agreement with Kárman-Tsien formula, calculations by Burago method approaches closer to value of the pressure-drop designated on Fig.3 as C_p^* which corresponds to the critical mode. Values of C_p^* were determined in work [18] and by the author and apparently, from Fig.3 they have almost coincided. On Fig.3 distribution of pressure-drop coefficient for an incompressible liquid also is shown. Comparison of results for compressible and incompressible flows specifies strong influence of the factor of compressibility.

In Fig.4(a) results of calculations for relative velocity on the cylinder obtained with the proposed approximate method, with a method of integrate relations [19], with a method based on application of Legendre transformation [12] and Rayleigh-Janzen method (see works [6], [8]) are compared. The results of calculations of T. Simisaki (1955) are borrowed from work of M. Holt and B. Messon [19]. Calculations were carried out for free stream Mach number $M_\infty = 0,37$, which is almost equal to critical Mach number (Table 1).



(a) (b)

Fig.4: Influence of Mach number on velocity at the surfaces of the cylinder



a) ellipse **b) spheroid**

Fig.5: Dependence of critical Mach number from a relative thickness of an ellipses and spheroids for water and air

Fig.4(a) shows a good agreement between the calculation results obtained with a method of integrate relations and with a method of application of Legendre transformations. The maximal value of velocity on the cylinder in a compressible flow calculated with the approximate method is larger than the appropriate values obtained by other methods, and approximately 1,25 times higher, than for an incompressible liquid. Fig.4(b) shows the plot of the maximal velocity on the cylinder versus Mach number of main flow. Results of calculations of Simisaki (see work [19]) with Rayleigh-Janzen method, M. Holt and B. Messon [19] with Dorodnitsyn method of integrate relations and Dombrovski calculations [9] with a method of approximation of an adiabat are shown there. The big difference in the results obtained with the same Rayleigh-Janzen method is caused by the fact that Simisaki obtained the solution with the fifth member of decomposition, while solution of Shih-I Pai [8] corresponds only to the first member of decomposition. From Fig.4(b) it follows, that the divergence of the calculated data increases with increase of Mach number of the main flow. As it is expected, the greatest mismatch of the results is marked at critical Mach number.

Fig.5(a), (b) shows the plots of critical Mach number versus relative thickness of an ellipses and spheroids for a mode of continuous flow of water and air. Fig.5(a), (b) specifies, that the same bodies in water, have smaller critical Mach number, than in air.

The effect of compressibility in water and in air is illustrated in Fig.6, which shows the relation of relative maximal speeds $\left(\bar{U}_{\max} = U_{\max}/U_{\infty}\right)$ for compressible and incompressible flow. From Fig.6 it follows, that influence of compressibility for water for the same speed is more, than for air. In Fig.7 results of calculations of pressure-drop coefficient for compressible and incompressible flow of water and air about 2-D and 3-D bodies are shown. From Fig.7 it follows that the effect of compressibility is more strongly shown in water, than in air. Compressibility has stronger effect at two-dimensional bodies, than at axisymmetric bodies.

CONCLUSIONS

The carried out calculations of flows of compressible liquid has shown, that the developed approximate method is exact enough and does not concede to more strict methods of the account of compressibility.

Movement of bodies in water with high speeds is characterized by smaller values of critical Mach numbers in comparison with movement of the same bodies in air.

At the same Mach number of flow at infinity compressibility in water is shown more strongly, than in air.

Compressibility is shown more strongly at a flow around two-dimensional bodies, than at a flow around axisymmetric bodies.

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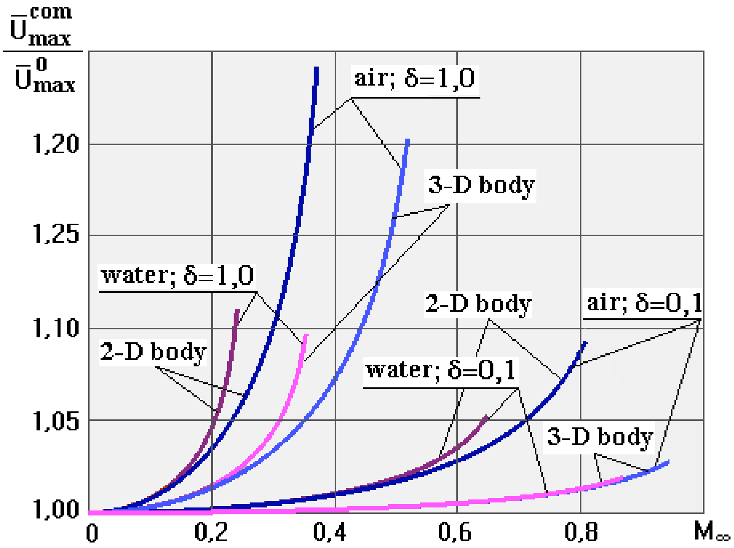
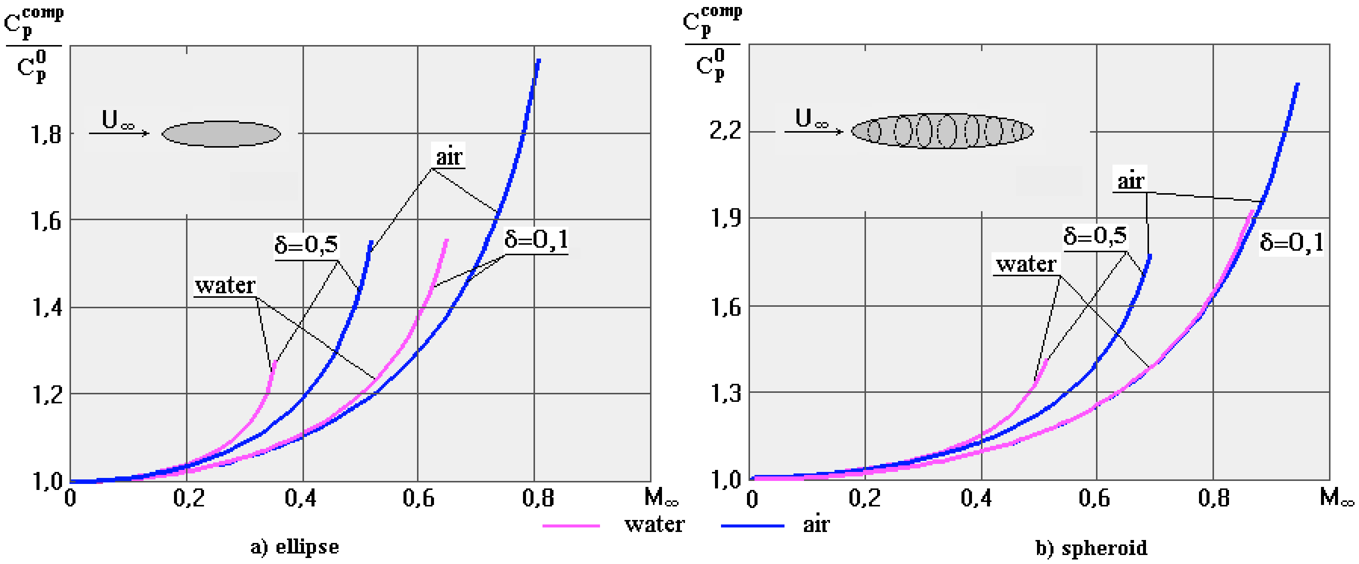


Fig.6: Maximal velocities at ellipse and spheroid versus Mach number for different medium



a) ellipse

b) spheroid

Fig.7: Influence of Mach number of compressible flow of water and air on pressure-drop coefficient at ellipse and spheroid

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