NUMERICAL PREDICTION OF CAVITATING FLOW OF LIQUID HELIUM IN A CONVERGING-DIVERGING NOZZLE

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ABSTRACT

The multiphase thermodynamic and hydrodynamic characteristics of the two-dimensional cavitating flow of liquid helium through a horizontal converging-diverging nozzle near the lambda point are numerically predicted to realize the further development and high performance of new multiphase superfluid cooling systems. First, the governing equations of the cavitating flow of liquid helium based on the unsteady thermal nonequilibrium multi-fluid model with generalized curvilinear coordinates system are presented, and several thermal flow characteristics are numerically calculated, taking into account the effect of superfluidity. Based on the numerical results, the two-dimensional structure of the cavitating flow of liquid helium though a horizontal converging-diverging nozzle is shown in detail, and it is also found that the generation of superfluid counterflow against normal fluid flow based on the thermomechanical effect is conspicuous in the large gas phase volume fraction region where the liquid to gas phase change actively occurs. Furthermore, it is clarified that the mechanism of the He I to He II phase transition caused by the temperature decrease is due to the deprivation of latent heat for vaporization from the liquid phase.

NOMENCLATURE

$e$ specific internal energy
$g^r_i$ contravariant vector of gravitational acceleration
$g^{ij}$ fundamental metric tensor
$h$ specific enthalpy

$J$ momentum flux density
$N$ number density
$p$ absolute pressure
$R$ radius of bubble or droplet
$S$ specific entropy
$T$ absolute temperature
$t$ time
$u^i, u^j$ contravariant velocity
$\alpha$ volume fraction
$\Gamma$ phase generation density
$\eta$ longitudinal coordinate
$\kappa$ ratio of specific heat
$\lambda$ thermal conductivity
$\mu$ dynamic viscosity
$\nu$ kinematic viscosity
$\xi$ transverse coordinate
$\rho$ density
$\nabla_i$ covariant differential

Subscripts
$(\text{ex})$ exit section of the duct
$(g)$ gas phase
$(i), (j), (k)$ contravariant component
$(i), (j), (k)$ covariant component
$(\text{in})$ inlet section of the duct
$(l)$ liquid phase
$(n)$ normal fluid
$(s)$ saturation
1 INTRODUCTION

Recently, the importance of the development of high performance cooling systems which can be employed under severe conditions, such as low temperature, microgravity, or the environment in space, has markedly increased, and fluid machinery systems using cryogenic fluid refrigerant are widely used in LNG (Liquefied Natural Gas) plants and aerospace technology [1, 2].

Among such fluids, liquid helium, which is known as the ultimate low temperature cryogen which possesses high functionality of zero viscosity with \( \lambda \) transition, is effectively utilized as a cooling device for superconducting magnets or infrared space telescopes and many other engineering applications [1]. When liquid helium is employed for cooling, cavitation frequently occurs in the flow duct, and the flow pattern consists of two phases. Thus, investigation of the cavitating flow characteristics of cryogenic fluids such as liquid helium is very interesting and important not only in the basic study of the hydrodynamics of cryogenic fluids [2], but also for providing solutions to problems related to new practical engineering applications.

However, the cryogenic system presently employed generally uses a refrigerant for the undercooling condition, and its cooling performance depends only on the single phase region. Thus, few attempts have been made to positively apply the extensive heat transfer and fluid acceleration characteristics of cryogenic two-phase flow to low temperature cooling systems [3].

Under the above-mentioned conditions, we contrived a new concept of a multiphase superfluid cooling system using liquid helium cavitating flow. The system can realize extensive low temperature cooling by utilizing the two-phase superfluid counterflow generated by He I to He II phase transition based on the occurrence of cavitation of the normal fluid, without the direct use of He II. Additionally, it is expected that the concept of multiphase superfluid cooling can be utilized for further development of available micro-cooling systems, such as MEMS (Micro-Electro-Mechanical Systems) technology using microbubbles [4], because the unique characteristics of zero- viscosity of superfluid working refrigerant prevents the frictional dissipation of capillary channels in microdevices. In order to develop a new type of superfluid cooling system and to estimate cooling performance numerically, we herein develop a new method for analyzing cavitating flow based on an advanced mathematical model, which takes the effect of superfluidity of the cavitating cryogenic flow state in the low temperature field into consideration.

In the present study, the two-dimensional thermal fluid characteristics of high-speed cavitating flow of liquid helium with phase change through the converging-diverging nozzle are numerically investigated. First, the governing equations of the cavitating flow of liquid helium based on the unsteady multi-fluid model are presented, and then several flow characteristics are numerically calculated, taking into account the effect of superfluidity.

2 Numerical Method

The system used in the numerical analysis is schematically depicted in Fig. 1. Applications using cryogenic fluid generally encounter obstacles, or complex pipe shapes such as those of an orifice or a converging-diverging section. Additionally, the transfer-tube for cryogenic fluids generally has many horizontal passage sections. Thus, the model used for analysis simulates the cavitating flow of liquid helium passing through a horizontal converging-diverging nozzle. The duct is filled with pressurized liquid helium. Flow immediately occurs when the outlet D-C is opened. Liquid helium is continuously introduced via the inlet section A-B, the flow is extensively accelerated at the point of the nozzle throat, and cavitation or liquid-to-vapor phase change is induced by a decrease of pressure.

2.1 Governing Equations

In the numerical model, the cryogenic cavitating flow state can be approximated to that of a homogeneous bubbly flow because the differences in the physical properties, such as density, viscosity and surface tension of the cryogenic fluid between the gas and liquid phases, are very small compared with those of the fluid at room temperature. The small difference in the properties between the gas and liquid phases is unique to cryogenic fluids. Accordingly, it seems reasonable to assume that the cryogenic cavitating flow pattern is easily formed in the bubbly two-phase flow. In the process of modeling, we consider the effects of superfluidity in two-phase liquid helium, namely, superfluid in He II and normal fluid in He I are treated as a perfect fluid and metastable fluid, respectively. In the calculation, we assume that the property of superfluidity appears when the fluid temperature becomes less than the \( \lambda \) point (temperature at normal fluid to superfluid transition, about \( 2.17 \) K); however, in the case of temperatures above the \( \lambda \) point, we assume that the superfluid behaves in the same manner as the normal fluid. Here, we consider only the temperature dependence of the superfluid and normal fluid densities; thus, the normal fluid-superfluid transition rate based on quantum theory is not strictly considered.

The calculation is carried out using the two-dimensional generalized curvilinear coordinate system \((\xi, \eta)\), with \( \xi \) and \( \eta \) denoting the transverse coordinate and the longitudinal coordinate, respectively. The model for analysis simulates the cavitating flow of liquid helium passing through the nozzle throat of the duct. In the numerical modeling under this condition, the following assumptions are employed to formulate the governing equations.
1. The cavitating flow is a two-dimensional unsteady duct flow.
2. The vapor gas phase is produced by the phase change of the normal fluid.
3. The energy exchange between the liquid and gas phases is taken into account.

For construction of the cavitating flow characteristics in the present numerical model, it is assumed that the gas phase is homogeneously dispersed in the surrounding liquid phase and that the flow structure will form a bubbly flow.

Under the above conditions, the governing equations of the flow are derived as follows.

The mass conservation equation for the gas phase is

\[ \frac{\partial}{\partial t} \left( \alpha_g \rho_g u_g^i \right) + \nabla_j \left( \alpha_g \rho_g u_g^j \right) = \Gamma_g^i. \]  

(1)

The mass conservation equation for the liquid phase is

\[ \frac{\partial}{\partial t} \left( \alpha_l \rho_l \right) + \nabla_j \left( \alpha_l J_l^j \right) = \Gamma_l, \]  

(2)

where the relationship \( \alpha_g + \alpha_l = 1 \) is assumed, and the liquid phase density, \( \rho_l \), must be comprised of a linear combination of the two components. The density is expressed by the sum of the normal fluid and superfluid components, and \( \rho_l \) is defined as: \( \rho_l = \rho_l(n) + \rho_l(s) \). For the two-fluid model, it is assumed that the entire temperature dependence of liquid helium densities enters through the variation of the normal fluid density. It is therefore possible to write

\[ \frac{\rho_l(n)}{\rho_l} = \begin{cases} \left( \frac{T_l}{T_{l0}} \right)^{5.6} & \text{for } T_l \leq T_{l0} \\ 1 & \text{for } T_l > T_{l0}, \end{cases} \]  

(3)

as the temperature dependence of the normal fluid density \([2]\). Because of this strong temperature dependence, the He II constitutes about 99% of the superfluid component at 1.0 K. The total densities of the two components, namely, the superfluid and the normal fluid densities in control volume are conservative in the numerical calculation process. Also, the liquid phase momentum flux density \( J_l^j = \rho_l(n)u_{l(n)}^j + \rho_l(s)u_{l(s)}^j \).

The combined equation of motion for a total gas and normal fluid is

\[ \frac{\partial}{\partial t} \left( \alpha_g \rho_g u_g^i + \alpha_l \rho_l u_{l(n)}^i \right) + \nabla_j \left( \alpha_g \rho_g u_g^j + \alpha_l \rho_l u_{l(n)}^j \right) = \quad \]  

\[ -g^{ij} \nabla_k p_l - \mu_l \nabla_j \nabla_k u_{l(n)}^i - \mu_l \nabla_j \nabla_k u_{l(s)}^i + \rho_l \rho_l \frac{\alpha_l^{(s)}}{2} g^{ij} \nabla_j \nabla_k u_{l(s)}^k \]  

\[ + \frac{1}{3} \left( \mu_l \nabla_j \nabla_k u_{l(s)}^k \right) g^{ij} + \alpha_l \rho_l \rho_l^{(s)} - \alpha_l F_l^{(n)} \]  

(4)

The combined equation of motion for a total gas and superfluid is

\[ \frac{\partial}{\partial t} \left( \alpha_g \rho_g u_g^i + \alpha_l \rho_l u_{l(s)}^i \right) + \nabla_j \left( \alpha_g \rho_g u_g^j + \alpha_l \rho_l u_{l(s)}^j \right) = \quad \]  

\[ -g^{ij} \nabla_k p_l + \alpha_l \rho_l S_l \nabla_j \nabla_k T_l + \alpha_l \rho_l \rho_l \frac{\alpha_l^{(s)}}{2} g^{ij} \nabla_j \left( u_{l(s)}^j - u_{l(s)}^l \right)^2 \]  

\[ + \alpha_l \rho_l \rho_l^{(s)} + \alpha_l F_l^{(s)} \]  

(5)

where the second terms on the right-hand side of Eqs. (4) and (5) denote the thermomechanical effect of the force based on the product of the entropy by the temperature gradient, and the third terms denote the effect of the momentum energy gradient based on the two-phase superfluid-normal fluid relative velocity caused by counterflow of the superfluid against the normal fluid. The terms mentioned above are peculiar to liquid helium with superfluidity \([2]\). The signs of these terms in Eq. (4) are opposite those in Eq. (5); thus, the forces based on the superfluidity of Eq. (4) act in the direction opposite those of Eq. (5). In this calculation, because the vapor phase is assumed to be produced...
by the phase change of the normal fluid, the cavitating flow of the superfluid is consists of the mixture flow of the vapor phase produced by the normal fluid and the superfluid. The term \( F_{\text{SN}} \) denotes the two-phase superfluid-normal fluid mutual friction interaction term based on the generation of vortex filaments in the superfluid [5, 6]. Additionally, \( \mu_T \) in Eq. (4) denotes the viscosity of the two-phase mixture flow that includes small dispersed bubbles. \( \mu_T \) was evaluated using the following formula by the viscosity of a suspension [7, 8]:

\[
\mu_T = \left[ 1 - \left( \frac{\alpha_g}{0.680} \right)^2 \right] \mu_{\text{SN}}, \quad (\alpha_g < 0.5)
\]

Eq. (6) being mainly applicable in the small gas phase volume fraction region. Concerning the viscosity, the present numerical model assumes that superfluid viscosity \( \mu_{\text{SN}} = 0 \) and that the dissipative interaction is due only to the normal fluid. This assumption corresponds to the physical fact that the superfluid experiences no resistance to flow and therefore no turbulence. The superfluid can flow through a duct without viscous drag along the boundaries. Equations (4) and (5) above are derived by complying the equations of momentum for both the gas and liquid phases.

To consider the effects of additional forces that act on the bubbles and radial expansion of the bubbles, the equation of motion for the gas phase is here replaced with the translational motion for the gas phase is here replaced with the translational motion from the steady state. In the above equation, the subscript \( m \) denotes the gas phase \((m=g)\) or liquid phase \((m=l)\). \( h_{\text{SN}}^{(i)} \) and \( h_{\text{SL}}^{(i)} \) are the enthalpy of the gas phase and the liquid phase at the interface, respectively. \( a^{(i)} \) is the interfacial area concentration. \( \Gamma_{B} h_{\text{SN}}^{(i)} \) and \( \Gamma_{I} h_{\text{SL}}^{(i)} \) are the interface energy transfer terms due to the liquid-vapor phase change. \( q_{\text{SN}}^{(i)} \) and \( q_{\text{SL}}^{(i)} \) are the heat transfer terms of mutual interaction between the vapor and liquid interface. \( q^{i} \) is the contravariant heat flow vector and \( \Phi \) is the energy dissipation function, as described below:

\[
\begin{align*}
q_{\text{SN}}^{(i)} & = -\lambda_{m} g^{ij} \nabla_{j} T_{m}, \\
\Phi_{m} & = -\frac{2}{3} \mu_{m} (\nabla_{i} u_{m}^{i})^{2} + 2 \mu_{m} s_{jm}^{i} s_{jm}^{i}, \\
s_{jm}^{i} & = \frac{1}{2} (\nabla_{j} u_{m}^{i} + \nabla_{i} u_{m}^{j}).
\end{align*}
\]

In the condition of the He II state, Gorter-Mellink 1/3 power law [2, 3] is considered to formulate the expression for \( \lambda \) in Eq (9) by the following equation.

\[
\lambda_{l} = \left( \frac{f^{-1}(T_{l})}{|\nabla_{j} T_{l}|^{3/2}} \right)^{1/3},
\]

where \( f(T_{l}) \) is the He II heat conductivity function which exhibits strong temperature dependence [2]. In Eq. (8), the mutual friction dissipation term is neglected because the energy transfer terms between the gas and liquid phases become dominant to the mutual friction dissipation term. Assuming that the mass of each vapor bubble and condensed liquid droplet in each computational cell is constant results in the following mass conservation equation for number density, \( N_{k} \):

\[
\frac{\partial}{\partial t} \left( \frac{4}{3} \pi R_{k}^{3} N_{k} \rho_{k} \right) + \nabla_{j} \left( \frac{4}{3} \pi R_{k}^{3} N_{k} \rho_{k} u_{m}^{j} \right) = \Gamma_{k},
\]

where subscript \( k \) denotes evaporation \((k = e)\) or condensation \((k = c)\). To close the governing equations, and to consider the expansion and contraction of bubbles, several additional constitutive equations are required [5].

### 2.2 Numerical Conditions and Procedure

To construct the numerical conditions for cavitation liquid helium flow, we refer to the previous experimental research on
the cryogenic cavitating internal flow condition of liquid helium [10–12]. The computational grid is generated referring to the geometry of a converging-diverging flow pipe which was used in a previous visualization measurement [10]. The finite difference method is used to solve the set of governing equations mentioned above. In the present calculation, the discrete forms of these equations are semi-implicitly obtained using a staggered grid. The grid is concentrated at the nozzle wall to capture the cavitation inception precisely. Then a modified SOLA (numerical Solution Algorithm for transient fluid flow) method of Tomiyama et al. [8], which is superior for the formulation and solution of a gas liquid two-phase flow problem, is applied for the numerical calculation. The Neumann type boundary condition is considered in the iteration process of the pressure correction equation, and the effect of void fraction is implicitly taken into account in each iteration process. The liquid phase velocity, \( u_l \), at the location of bubbles is calculated using an area-weighting interpolation method which was used in the SMAC algorithm by Amsden and Harlow [13].

To determine the boundary conditions, nonslip conditions for prescribed normal fluid velocities and free-slip conditions for prescribed superfluid velocities are applied to the sidewalls, A-D and B-C, in Fig. 1. Also, a fully developed velocity profile is applied for normal fluid velocities to the inlet cross-sectional area of the flow duct, A-B. A convective outflow condition is applied for normal fluid and superfluid velocities to the exit section of the duct, D-C. Adiabatic conditions are applied for thermal boundary conditions at the duct wall surface. The initial stationary condition of the liquid phase is assumed to be the pressurized He I state. Also, the initial conditions at the inlet section of the flow duct are given as; Inlet pressure: \( p_l^{(in)} = 0.20 \text{ MPa} \), Outlet pressure: \( p_l^{(out)} = 0.101 \text{ MPa} \), Internal energy: \( e_l^{(in)} = 6.021 \text{kJ/kg} \), Inlet width of duct: \( D = 10.0 \text{mm} \). For other physical properties used in constitutive equations, \( \mu_l \) and \( S_l \) are given as functions of temperature [14]. The constitutive equation for the two-phase heat transfer coefficient is given as a function of temperature based on the previous experimental results in reference [3].

### 2.3 Results and Discussion

Figure 2 shows the numerical results of the transient evolution of the void fraction \( \alpha_g \) contour, Fig. 4 shows the instantaneous liquid phase pressure \( (p_l) \) contour, and Fig. 3 shows the transient evolution of the liquid phase temperature \( (T_l) \) contour. The direction of flow is left to right. As shown by Fig. 2, when the exit section is opened instantaneously, pressurized liquid He I flows into the converging section of the duct at high speed and is further accelerated by the decrease of the cross-sectional area, and \( p_l \) locally decreases in the nozzle throat section. It is clear that the phase change effectively occurs in the downstream of nozzle throat section and that a cloud cavity which consists of concentrated small bubbles is formed in the wall surface vicinity of the throat section. In addition, it is found that the cloud cavity is especially formed and grows on the upper wall surface of the diverging throat section due to the influence of the buoyancy which acts on the bubbles and the shear force which simultaneously acts on the bubbles in contact with the wall. Because the buoyancy acts on the bubbles, there is a tendency for the bubbles to migrate and aggregate on the upper wall surface. Thus, the void fraction profiles become asymmetric. As time \( t \) elapses, cavitation inception effectively occurs and the cloud cavity grows alongside the passage wall surface. When the magnitude of the cavity is above a certain size, the cavity is detached from the cloud, and it remains in the high volume fraction region as the gas phase moves downstream. Furthermore, with the elapse of time, the cloud cavity accompanying evaporation and condensation exhibits convective and dissipative behavior downstream of the diverging nozzle throat, and the gas phase spreads throughout the inner flow duct because of the decrease in the slip ratio and the gas phase velocity resulting from the sudden change of both longitudinal and transverse pressure gradients. As a result, it is found that a homogeneous profile of \( \alpha_g \) is formed over the downstream duct.

As shown by Figs. 2 and 4, the void fraction \( \alpha_g \) has a large value in the region where it is close to the cavity center, because the pressure gradient in the cloud cavity has a distribution which becomes negative from the contour of the cavity toward the center, and because the ratio where the bubbles accumulate increases as the position approaches the cavity center. Additionally, the expansion effect of bubbles becomes larger. Furthermore, in the region of the high volume fraction of the gas phase, the pressure distribution changes markedly because of the normal fluid-superfluid transition due to the momentum terms in Eqs. (4) and (5) that include the thermomechanical effect term, the momentum energy gradient term based on relative superfluid-normal fluid velocity, and the superfluid-normal fluid mutual friction interaction term. In this numerical calculation, it is assumed that the existence of the large gas phase volume fraction region indicates that the small size bubbles shown in Fig. 5 constitute a closely aggregated region and that the downstream flow state maintains a very closed bubbly flow in the large void fraction region. The bubbles are concentrated toward the center of the vortex due to the negative pressure gradient in the vortex.

Next, to confirm the validity of the numerical results, the present results on the void fraction profile are compared with the previous visualization measurement of cavitating flow of liquid He in a converging-diverging pipe which has the same geometry as that of the present computational nozzle shape [10]. The initial conditions for this experiment are generally similar to the present numerical condition. However, because the experiment is conducted making use of only vacuum insulation, initial temperature of the working fluid is considered to be higher than the \( \lambda \) point, and quantitative comparison with the present result is
difficult. According to these results, it is both numerically and experimentally found that cavitation inception occurs on the upper wall surface of throat. It is also found that the numerical results of the detachment and development of the cloud cavity, diffusion of the gas phase, and the time dependent profiles of void fraction show qualitative agreement with the results of the visualization measurement.

Focusing on Figs. 2 and 3, in the large $\alpha_g$ region in the vicinity of the wall surface from the position of the cloud cavity contour downstream of the throat where the vapor phase change actively occurs, it is found that the phase transition from He I to He II is generated ($\lambda$ transition) and that it conspicuously exhibits the characteristics of superfluidity. The effect of superfluidity with He I to He II phase transition is mainly caused by the decrease in liquid phase temperature or internal energy due to the deprivation of latent heat for vaporization from the liquid phase and to the change of the specific heat of the liquid phase with the change of pressure gradient. From Fig. 3, it is especially found that the temperature around the interface between the large gas phase volume fraction region and the liquid phase region decreases with the increase in the phase change. The liquid phase temperature decrease due to the latent heat or the energy exchange between liquid- and vapor phase in the vaporization process is
characterized by the interfacial energy transfer terms with the phase change in Eq. (8). With time, the profile of the liquid phase temperature $T_l$ gradually becomes homogeneous due to the effect of temperature diffusion and gas phase condensation. The tendencies of those numerical results for temperature decrease with the He I to He II phase transition show qualitative agreement with the experimental datum on the He I cavitation in the saturated condition by Ishii and Murakami [11, 12].

Figure 5 shows the fluctuation of bubble radius, $R_g$, as a function of the time at position E (as depicted in Fig. 1) just downstream of the nozzle throat, where the cavitation actively occurs. From Figs. 2-5, it is clarified that the decrease of $p_l$ induces an increase of $\alpha_g$ and that the expansion or contraction of bubble radius $R_g$ corresponds to the change of $p_l$. However, the displacement magnitude of $R_g$ has a small value. Thus, it is also clarified that the generated cavitation bubbles maintain a small size in the vaporization process and in the initial cavitating flow state.

Figures 6 and 7 show profiles of the liquid phase normal fluid velocity component $u_{i(n)}^{l}$ and the superfluid velocity component $u_{i(s)}^{l}$ around the nozzle throat, respectively. The flow separation and backward-flow of $u_{i(n)}^{l}$ locally occur in the vicinity of the wall of the throat section upstream of the cavitation inception point. From comparison of Figs. 6 and 7 with Figs. 2 and 3, superfluid counterflow against the normal fluid flow is conspicuously found in the large void fraction region where the cavitation is actively generated because of the increase in momentum exchange between the gas and liquid phases. With elapse of time $t$, vortices of normal fluid and superfluid are formed and advected downstream of the throat. Due to slight viscosity of normal fluid, the vortices are slightly different in shape. The superfluid counterflow against normal fluid is mainly caused by the momentum terms in Eqs. (4) and (5), i.e., the temperature gradient term (thermomechanical effect), and the momentum energy gradient term for superfluid-normal fluid relative velocity.

3 Conclusion

When the cavitation of He I is generated, the characteristics of superfluidity with $\lambda$ transition are conspicuously found surrounding the cloud cavity and in the large gas phase volume fraction region where the unsteady cavitation actively occurs. Also, the effect of superfluidity with He I to He II phase transition is mainly due to the decrease in liquid phase temperature or internal energy due to the deprivation of latent heat for vaporization from the liquid phase.

The generation of the superfluid counterflow against normal fluid caused by the momentum terms based on superfluidity was conspicuously found when the vaporization with He I to He II phase transition occurs.
Figure 7. Instantaneous superfluid velocity vector

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