ABSTRACT

Barotropic models despite their attractiveness, due to their simplicity and their clear physical meaning, need to be appropriately set by defining a key parameter that is the value of minimum speed of sound (SoS) for the liquid-vapor mixture. The different value of SoS setting can significantly affect the flow field resulting from numerical simulation especially in terms of unsteady modeling. This paper, hence, is addressed to investigate the influence of the minimum speed of sound of the mixture on the simulation of unsteady cavitating flow by using a barotropic model. In particular, results for four cases of cavitating flow around a NACA0015 airfoil at 8° angle of attack are reported, and a barotropic relationship is implemented to take into account the liquid-vapor phase change. The variations of the flow dynamic response and the variations of the main flow features associated with different values of minimum speed of sound are reported and discussed.

Results show that the minimum speed of sound of the mixture plays a very important role in the numerical simulation of cavitating flow revealing that a sudden change of flow field response occurs when a threshold value of minimum speed of sound is reached.

INTRODUCTION

The difficulty in the numerical modeling of cavitating flows using both an one-fluid (homogeneous) approach (Delannoy and Keuny, 1998, Kubota et al., 1992, Avva and Singhai, 1995, Chen and Heister, 1994(a), Pascarella et al., 2000,2001 and Salvatore et al. 2001) and a two-phase approach (Chen and Heister, 1994(b), De Jong and Sabnis, 1991) is well known. This paper deals with a numerical method belonging to the first family and, in particular, reports a work on the simulation of unsteady homogeneous cavitating flows in thermal equilibrium conditions.

A cavitating fluid often corresponds to a complicated situation from a numerical point of view; in fact, both incompressible zones (pure phases) and regions where the flow may become highly supersonic (liquid-vapor mixtures) are present in the flow field and need to be resolved. The numerical stiffness of the phenomenon is further increased both from the high density ratio between the two phases (for water at 293 K the density ratio is equal to 1.7 x 10^-5) and from the strong shock discontinuity occurring at re-condensation. This singular behavior is due to the huge variation of speed of sound as the flow goes from a pure phase to a mixture. For instance, in the same hypothesis indicated above, the speed of sound is equal to about 1500 m/s for the liquid phase, 400 m/s for vapor phase and 3.2 m/s for a liquid-vapor mixture corresponding to a value of void fraction equal to 0.5 (Jacobsen, 1964). Moreover, the latter value of minimum speed of sound is calculated in thermal equilibrium conditions and neglecting both the exchange of mass between the two phases and the surface tension effects. Indeed, the calculation of minimum speed of sound can vary largely depending on the hypothesis that one assumes. For example, using the steam tables (Rivkin, 1988) and assuming cavitation as an isenthalpic transformation (Avva, 1995), the minimum speed of sound at 293 K is approximately equal to 0.004 m/s.

In addition, the speed of sound is a parameter that can be easily defined only when the thermodynamics of phase change is very fast or very slow (frozen - equilibrium) with respect to the characteristic time of the phenomenon (Brennen, 1995). For all
the other cases a speed of sound cannot be simply defined. This paper will not deal with the physical uncertainty in the definition of the correct speed of sound, which is still an open problem.

The present research focuses on the flow response to speed of sound variation and on the capability of the numerical method to capture the main relevant aspects of cavitating flows. Computations were made on a two-dimensional flow field around a NACA0015 airfoil at an angle of attack of 8 degrees, assuming inviscid flow.

NOMENCLATURE

\[ a = \text{speed of sound} \]
\[ C_p = \text{pressure coefficient} \]
\[ K = \text{cavitation index (Rouse and McNown)} \]
\[ p = \text{pressure} \]
\[ p_r = \text{total pressure} \]
\[ T = \text{temperature} \]
\[ t = \text{time} \]
\[ \dot{x} = \text{vapor production} \]
\[ V = \text{velocity vector} \]

Greek

\[ \alpha = \text{thermal diffusivity, void fraction} \]
\[ \varepsilon = \text{phase fraction} \]
\[ \mu = \text{dynamic viscosity} \]
\[ \rho = \text{density} \]
\[ \sigma = \text{cavitation number} \]
\[ \Delta {P}_{\text{max}} = \text{maximum value of pressure correction} \]
\[ \Sigma = \text{thermal parameter} \]

Subscripts

\[ \infty = \text{free stream condition} \]
\[ L = \text{liquid; pure liquid in the barotropic two-phase representation} \]
\[ G = \text{gas; pure vapor in the barotropic two-phase representation} \]
\[ \text{re} = \text{reference} \]
\[ S = \text{saturation} \]
\[ V = \text{vapor} \]

CAVITATING FLOW MODEL

All the past main efforts to numerically model cavitating flows can be roughly divided into two classes: interface tracking and continuum methods.

In the first case, each phase is treated separately and, typically, there are separate balance equations for each phase. The procedure starts with a tentative surface (a tentative interface between two phases) that during the calculation is iteratively updated. The interface tracking method has been successfully applied to 2-D flows (Brewe, 1986, Vijayaraghavan and Keith, 1990, Deshande and Merkle, 1979, and Chen and Heister, 1994(b)). These methods present two main disadvantages: they are computationally expensive, due to the double number of balance equations, and they cannot be easily extended to 3-D flows because of the obvious difficulties involved in tracking complex interfaces in space.

Continuum methods treat the flow as a homogeneous mixture employing a “void fraction” variable to quantify the intensity of cavitation; this variable is defined as the ratio between the vapor volume and the whole volume (liquid plus vapor). Continuum methods can be sub-divided into three categories: analyses introducing a vapor source production term, analyses including the bubble dynamics and analyses assuming a barotropic state law. The model that employs a vapor production equation (Song and He, 1998) resolves, together with the momentum and continuity equations for the mixture, the following phasic continuity equation where \( p < p_v \):

\[
\frac{Dp}{Dt} = S = C_v (p - p_v) . \quad (1)
\]

This approach imposes the choice of the empirical coefficient \( C_v \) that, unfortunately, depends on several parameters such as properties of the fluid, body geometry and other flow conditions. Furthermore, being the numerical solutions very sensitive to the choice of \( C_v \), the use of this method seems to be confined only to academic purposes.

Kubota et al. (1992) proposed one of the earliest continuum methods. In his approach he treats the cavitating flow as a uniform mixture of liquid and very little bubbles. The mathematical formulation of the problem is very rigorous: the growth and the collapse of bubble clusters is taken into account by the introduction of a modified Raleigh’s equation. Also this method has free, user-supplied constants, that are the initial radii and initial number density of the bubbles. Another limitation of the method proposed by Kubota et al. (1992) is represented by the severe stability problems encountered at low void fractions, which makes this approach not easily applicable to any cavitating flow.

Analyses in the third category assume the existence of an effective barotropic relation for the homogeneous mixture. The barotropic relation \( p = f(\rho) \) assumes that the fluid pressure is a function of fluid density only. This implies that all the effects caused by bubble content are disregarded except for the compressibility and that the bubbly mixture can be regarded as a single-phase compressible fluid.

Usually, the methods employing a barotropic state law, such as that proposed here, assume a continuous variation of density between liquid and vapor values in a range of pressures centered at the vapor pressure.

Such a variation can be represented by a sinusoidal curve (Delannoy and Keuny, 1998), by a polynomial curve (Song and He, 1998), etc. In all cases, the maximum slope of the curve occurs at the vapor pressure. For the present study a simple sinusoidal law as represented in Fig. 1 has been chosen, where \( a_{\text{min}} \), the minimum speed of sound, is the parameter that determines the maximum slope of the barotropic curve.

The following relation represents the barotropic variation in the two-phase zone:

\[
p = \rho_v + \frac{1}{a^2_{\text{min}}} \left( p_l - p_v \right) \sin \left( \frac{p - p_v}{p_l - p_v} \pi \right) \quad (2)
\]

where
\[ \rho_v = \frac{\rho_1 + \rho_v}{2} \]

\[ p_l = p_v + \left( \rho_1 - \rho_v \right) a^2_{min} \frac{\pi}{2} \]

\[ p_g = 2p_v - p_l \]

The density derivative with respect to pressure is

\[ \frac{1}{a^2} = \frac{dp}{d\rho} = \frac{1}{a^2_{min}} \cos \left( \pi \frac{\rho - \rho_v}{p_l - p_v} \right) \]  

(3)

And so one finds that the speed of sound goes to infinity for \( p = p_v \) and for \( p = p_g \) (incompressible regions) and is equal to \( a_{min} \) for \( p = p_l \).

![Fig. 1 Barotropic law](image-url)

**Fig. 1 Barotropic law**

For this approach, it is necessary to specify three constants: the minimum speed of sound of the mixture, the liquid density and the vapor density. For the last two properties one can choose the density values corresponding to the equilibrium conditions (for fixed temperature); the choice of the opportune value of \( a_{min} \) is the main difficulty and the study of its effects on the cavitation simulation is just the aim of the present work.

A reasonable value of \( a_{min} \) could be assumed within the range limited by the values obtained following either the so called dynamic approach of Jaksen (1964) or the isenthalpacial transformation approach proposed by Avva (1995). These two approaches give very different values of minimum speed of sound.

Following the dynamic approach, the mass and heat transfer between the two phases is neglected. The thermodynamic transformation is assumed to be isentropic for each phase. In the two hypotheses the speed of sound in the homogeneous flow is

\[ \frac{1}{a^2} = \frac{1}{a^2_L} \left( \frac{\rho_v}{\rho_L} \right)^\alpha + \frac{1}{a^2_v} \left( 1 - \alpha \right) \frac{\rho_L}{\rho_v} + \frac{1}{a^2_L} \left( \frac{\rho_v}{\rho_L} \right)^\alpha \left( 1 - \alpha \right) \frac{\rho_L}{\rho_v} \]  

(4)

where \( \alpha \) is the void fraction, defined as

\[ \alpha = \frac{1 - \rho / \rho_L}{1 - \rho_v / \rho_L} \]

In the isenthalpacial transformation, the two phases are assumed in thermal equilibrium. Using the thermodynamic tables (Rivkin, 1988), it is possible, if a value of enthalpy, \( h \), is fixed to establish a correspondence between pressure and density for the homogeneous mixture, therefore defining a barotropic relationship from which the speed of sound can be easily derived.

**NUMERICAL APPROACH**

In the homogeneous approach and in the hypothesis of thermodynamic equilibrium the governing equations to solve are the Navier Stokes equations written in the unsteady, compressible form for one phase without the energy conservation equation; the barotropic state law completes the set of equations. The model may be written as three partial differential equations and one algebraic equation of state:

\[ \frac{\partial p}{\partial t} + \nabla \cdot \rho V^2 = 0 \]  

(5)

\[ \frac{\partial \rho}{\partial t} + \nabla \cdot \rho V^2 = - \frac{\partial p}{\partial x} + V \cdot (\mu \nabla V) \]  

(6)

\[ \frac{\partial \rho V}{\partial t} + \nabla \cdot \rho V^2 = - \frac{\partial p}{\partial y} + V \cdot (\mu V) \]  

(7)

\[ \rho = \rho(p) \]  

(8)

where \( \rho \) and \( \mu \) are, respectively, the density and the viscosity of the mixture. The barotropic law of state, Eq. (8), defines the value of the density; a simple and stable way to calculate \( \mu \) is the following (Kubota et al., 1992):

\[ \mu = \mu_L \cdot \alpha + \mu_v (1 - \alpha) \]  

(9)

where \( \mu_L \) and \( \mu_v \) are the vapor viscosity and the liquid viscosity respectively, and \( \alpha \) is the void fraction. The choice of the numerical method to approach cavitation is very critical. In fact a cavitating flow presents huge gradients at the liquid-vapor interface. Furthermore, the flow field goes from incompressible in the one-phase regions to highly supersonic where a liquid-vapor mixture is present. Thus, the main requirements for the flow solver are the capability to work well in a wide Mach number spectrum, from zero to Mach>>1, and the shock-capturing or interface-capturing property in order to solve correctly the liquid-vapor surface discontinuity.

The most suitable methodology to solve the governing equations seems to be a pressure-based method with a finite volume discretization written in conservative form, based on the SIMPLE (Semi Implicit Method for Pressure Linked Equations) algorithm (Patankar, 1980).

The code developed in the present work, named ALL2D, can be applied to both plane and axi-symmetric configurations; under steady or unsteady conditions, and can be used to simulate both inviscid and viscous laminar flows. In the developed code a non oscillatory TVD 2nd / 1st order scheme, derived from SMART (Gaskell, and Lau, 1988), is adopted.

ALL2D works with a body-fitted grid; the governing Eqs, (5), (6) and (7) have been reported here only in a Cartesian coordinate system for simplicity; in any case, applying the tensor rules, the system of Eqs, (5)-(7) can be transformed in an equivalent system written in a curvilinear reference frame. The actual form of the governing is more and can be found in Kadja and Leschziner (1987).
As in all the methods derived from the SIMPLE algorithm, also in ALL2D it is necessary to execute the so-called under-relaxation at the end of each iteration to promote stability; the value of the general variable $\phi$ at location $L$ may be expressed by:

$$\phi_L^{\text{new}} = \alpha \phi_L^{\text{old}} + (1 - \alpha) \phi_L^{\text{new}}$$  \(10\)

where $\alpha$ is a value between zero and one. Particular attention must be paid when the flow is cavitating; in this case, to avoid abrupt oscillations of density, the under-relaxation of pressure must be limited by imposing values to the pressure correction always less than $(p_l - p_v)$. A way to achieve this is to introduce a varying under-relaxation factor which depends on the pressure correction itself as follows:

$$\alpha_p = \frac{1}{\Delta P_{\text{max}}} \chi$$  \(11\)

where $\Delta P_{\text{max}}$ is the maximum value of pressure correction and $\chi$ is an arbitrary number greater than one.

VALIDATION

The numerical method has been validated using the experimental data available from Rouse and McNown (1948), consisting in pressure measurements on the surface of some head forms at zero angle of yaw in water under steady state conditions. Although these experiments are very old the data are quite reliable. In general, it is very hard to find reliable experimental data on cavitating flows. Among the several tests reported in Rouse and McNown (1948) two different head form shapes have been chosen. Both head forms belong to the so-called “Rounded Series”; in particular, they are called “2-Caliber Ogival” and “Hemispherical”, Fig. 2. The comparison between numerical and experimental data has been made for each geometry at two different flow conditions corresponding to incipient cavitation and fully developed cavitation, see Tab.1. Following the terminology adopted in Rouse and McNown (1948) a cavitation index $K$ has been defined as:

$$K = \frac{P_c - P_l}{P_0 - P_l}$$  \(12\)

The simulations have been performed neglecting the viscous effects (Euler approximation) and assuming a minimum speed of sound of 0.12 m/s. The latter value has been chosen after an accurate tuning phase. In particular, the assumed minimum speed of sound is the value that leads to the best matching between computation and experimental data in the fully developed cavitation for the 2-Caliber Ogival test case; this value is maintained constant for all the other tests.

The calculations have been carried out assuming a temperature of 320 K; the corresponding density ratio is equal to 13305 (989/0.072) and the vapor pressure is equal to 10530 Pa. Both the very high density ratio and the very low speed of sound represent quite severe conditions: in most of the works in the literature on numerical modeling of cavitation using a barotropic law of state, a maximum value of density ratio of 1000 and a minimum speed of sound of 0.5-1 m/s is reported in Delannoy and Keuny (1998) and Hoeijmakers et al. (1998), for example. For all simulations the total pressure at inlet and the static pressure at outlet have been imposed as boundary conditions. The comparison between calculated and measured $C_p$ distributions along the surface of the ogival head form is shown in Fig. 3 for the case of incipient cavitation, as shown by the small zone where the $C_p$ remains almost constant. A good agreement between computed and experimental values is attained. This agreement is still good for the fully developed cavitation case, as shown in Fig. 4: the length of the vapor bubble has been captured rather well. The discrepancy in the re-condensation zone, where the computation shows a shock, is probably due to thermal non-equilibrium effects.

The most severe case is represented the fully developed cavitation on the hemispherical head form. Also for this case, agreement is excellent up to the end of the cavity where some discrepancy can be noted at re-condensation, Fig. 6.

Tab. 1 Validation tests

<table>
<thead>
<tr>
<th>Headform</th>
<th>Headform</th>
</tr>
</thead>
<tbody>
<tr>
<td>K=0.3 (incipient cav.)</td>
<td>K=0.7 (incipient cav.)</td>
</tr>
<tr>
<td>K=0.2 (fully developed cav.)</td>
<td>K=0.2 (fully developed cav.)</td>
</tr>
</tbody>
</table>

Fig. 2 2-Caliber and Hemispherical Head Form

Fig. 3 2-Caliber Ogival Headform, $K=0.3$
RESULTS

The goal of this work is to investigate the change in the dynamic response of the cavitating flow field to a variation of the minimum speed of sound, $a_{\text{min}}$. Water at temperature of 323 K, that corresponds to an enthalpy equal to 200 kJ/kg, has been chosen as working fluid.

Under these conditions, the isenthalpic model (Avva, 1995) and the dynamic approach (Jackobsen, 1964) give the following values:

- Isoenthalpic model $a_{\text{min}} = 0.17$ m/s
- Dynamic approach $a_{\text{min}} = 8.16$ m/s

A parametric study of the cavitating flow for a variation of minimum speed of sound between the above two limits has been performed. In particular four cases have been investigated: $a_{\text{min}}=3.0$ m/s, 2.0 m/s, 1.0 m/s and 0.8 m/s. As it will be shown, these values are representative of two different extreme behaviors, i.e. sheet cavitation with a quasi-steady field ($a_{\text{min}}=3.0$ m/s) and an unsteady sheet cavitation formation with a very fast variation of both bubble size and cavitation intensity ($a_{\text{min}}=0.8$ m/s).

Calculations have been performed assuming a Naca0015 airfoil at 8 degrees of incidence as test case. The computational domain has been discretized by means of a C-grid; three grid levels have been initially used for case a, consisting in 81x31, 121x46 and 162x61 grid points, respectively. Only small variations were observed in the flow field obtained with the three grids. No specifics grid independency analysis was performed for the other cases because of the very long computational times. All runs were performed with the 162x61 grid points. A close up view of the grid in shown in Fig. 7.

The cavitation number is a characteristic parameter to identify a similarity for cavitating flow:

$$\sigma = \frac{p_{\text{ref}} - p_{\text{L}}}{0.5 \rho_{\text{ref}} V_{\text{ref}}^2}$$

where reference values are specified at inlet of the computational domain.

Table 2 summarizes the conditions for the four cases performed:

<table>
<thead>
<tr>
<th>Case</th>
<th>Min. speed of sound [m/s]</th>
<th>Inlet velocity [m/s]</th>
<th>Time step [s]</th>
<th>Integration time [s]</th>
<th>$\sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>3.0</td>
<td>3.0</td>
<td>0.02</td>
<td>~7.0</td>
<td>~1.0</td>
</tr>
<tr>
<td>b</td>
<td>2.0</td>
<td>3.0</td>
<td>0.02</td>
<td>~7.0</td>
<td>~1.0</td>
</tr>
<tr>
<td>c</td>
<td>1.0</td>
<td>3.0</td>
<td>0.01</td>
<td>~55</td>
<td>~17.0</td>
</tr>
<tr>
<td>d</td>
<td>0.8</td>
<td>3.0</td>
<td>0.01</td>
<td>~17</td>
<td>~17.0</td>
</tr>
</tbody>
</table>

Tab. 2 Run matrix
The thermodynamic reference values have been derived from the thermodynamic tables (Keenan, 1969) and are reported in the Tab. 3.

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temperature</td>
<td>323 K</td>
</tr>
<tr>
<td>Liquid density</td>
<td>$\rho_l$ 989 kg/m$^3$</td>
</tr>
<tr>
<td>Vapor density</td>
<td>$\rho_v$ 0.083 kg/m$^3$</td>
</tr>
<tr>
<td>Vapor pressure</td>
<td>$p_v$ 12352 Pa</td>
</tr>
<tr>
<td>Liquid sound speed</td>
<td>$a_l$ 1542 m/s</td>
</tr>
<tr>
<td>Vapor sound speed</td>
<td>$a_v$ 443 m/s</td>
</tr>
</tbody>
</table>

Tab. 3 Thermodynamic properties of water-vapor equilibrium mixture

In Fig. 8, a representation of pressure measured at a point near the inlet as a function of time for the case a is shown. There is an initial decrease corresponding to the ramping of the outlet pressure (of 4 seconds) and then after a quite brief oscillation a steady value of pressure is reached.

In Fig. 9 a representation of density of liquid-vapor mixture and then the extension of cavitating zone is shown; for this case, a minimum value of density of about 507 kg/m$^3$ is reached. The recondensation occurs, in the zone of high intensity of cavitation, by an almost normal shock; this is clearly visible by the pressure representation, Fig. 10, where a strong compression can be noted.

Following the above analysis for the case b, also for this value of minimum speed of sound a steady-state and a stationary size and location of cavitating zone is reached. The dimension of region involved in phase changing is larger than in the previous case, Fig. 11, and the minimum value of density is decreased up to 346 kg/m$^3$. The shock, as shown by Fig. 12, is stronger than previous case.

For case c, a steady state value is reached only after a long time pressure oscillation, Fig. 13. The minimum density value is lower than previous, Fig. 14, decreasing to 138 kg/m$^3$. Cavitation region is about 60% of chord length. The recondensation occurs by a shock wave that is quite normal to airfoil surface, Fig. 15.

The variation of pressure measured as a function of integration time for the case d, with speed of sound equal to 0.8 m/s is shown in Fig. 16. Unlike previous cases there is not oscillation dumping of pressure. This value of minimum speed of sound can be assumed as a value after which there is not dumping for the examining configuration. The meaning of this result is that the higher values of $a_{min}$, the greater is the steadiness of flow field; furthermore for the assigned flow there is an $a_{min}$ value that divides a steady from an unsteady flow response. These results could not be extended to other flow field but they give useful information when a value of $a_{min}$ must be assumed. For instance, if the results of a numerical simulation are representative of a steady flow field and the experiments show that this is not true, one of the reasons could just be due to the choice of a too high value of $a_{min}$. Fig. 16 shows also a periodical behavior with an almost regular period of 1.1 s; the amplitude is not as regular as frequency and its value can be approximately assumed equal to the ten percent of exit value of pressure. Being this case not steady the results depends on the assigned initial flow field. For this case several attempts have been made to reach a good value of convergence and accuracy. The results shown regard a case initialized with a steady flow field corresponding to minimum speed of sound of 1 m/s. Other attempts such as those used for the cases with higher speed of sound, that is the slow decompression ramp technique, have shown severe accuracy and convergence problems and in each cases the resulting flow fields were very different from the one here presented. In Fig. 17 are shown density contour plot and the streamlines regarding four different instants of computation. By this representation the reason of unsteadiness can be understood because a periodical formations and destroying of a vortices is clearly shown. This mechanism is the same as that of reentrant jet. In the latter case, that occurs at higher values of streamline curvature around the cavitation zone (lower values of cavitation numbers), the vortices goes inside the main bubble breaking it into two or more parts. The trend of liquid phase to invade the two-phase zone is observable by means of the density contour plot representation of Fig. 18.
A comparison among cavitation region extension is shown in Fig. 19. The rising of extension of two-phase zone with a speed of sound decreasing is evident for the first three cases reaching a limit length of about 2/3 of cord extension when the flow field becomes unsteady.

For these test cases reliable quantitative measurements are not available for comparison. However, the qualitative flow behavior is reported and discussed in Kubota et al. (1992): the vapor bubble evolution is the same as that computed in the present work.
CONCLUSIONS AND FURTHER DEVELOPMENTS

In this paper a study on the numerical modeling of unsteady cavitation is presented. A barotropic cavitation model is implemented within a 2-D planar and axi-symmetric Navier–Stokes flow solver. To validate the model the simulations of four steady test cases are shown. For all the flow field configurations the results are in good agreement with the available experimental data. The only discrepancies of the calculations are located in proximity of the re-condensation zone.

The variation of response of cavitating flow around a NACA0015 airfoil for a variation of minimum speed of sound has been investigated. Four computations have been executed varying the speed of sound inside a representative range. The extremes of that range were included between the values of SoS calculated with the dynamic model and the isenthalpic model.

Two main conclusions can be drawn. First, at equal cavitation number, the flow steadiness decreases with the diminution of speed of sound and the field goes from a quasi-steady flow for the case with a minimum speed of sound of 3 m/s to an almost periodic response for the case at minimum speed of sound of 0.8 m/s.

The other, significant conclusion is that the dimension of cavitation region, in the case in which the flow is steady or quasi-steady, tends to decrease at higher values of $a_{\text{min}}$.

For all the calculations performed, due to the stiffness of the phenomenon, very low under-relaxation parameters and very low time step increments, and therefore a large amount of computer time have been needed.

In terms of physical modeling, the possibility to introduce more accurate forms of the law of state or new differential equations within ALL2D to take into account non-equilibrium effects will be in the future evaluated. The possible implementation of new equations will be justified only by a better understanding of the cavitation phenomenon and its interactions with the other effects, such as turbulence. The huge difficulty to improve the understanding of such phenomena justifies once more the simplicity of the assumptions adopted in this work for the state law and the Euler approximation. Despite these approximations, the results obtained at the present stage of this project are surprisingly encouraging.
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REFERENCES


