TEMPORAL EVOLUTION OF THE FREQUENCY OF A BUBBLE CHAIN

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ABSTRACT

A chain of bubbles has acoustic properties different from the classic Rayleigh-Plesset theory for isolated bubbles. Data from experimental measurements taken from the acoustic field emitted by a bubble chain are reported in this paper. Smoothed Pseudo Wigner-Ville (SPWV) distributions were used to examine the temporal evolution of the energy of the resonant frequency in the domain around the bubble chain. Pressure spectra under different conditions are also reported. It is shown that there is a significant time delay of the resonant-frequency energy in the direction along the chain. Analysis of the temporal evolution of the frequency may be useful for the diagnosis of multiple-bubble interactions in discrete systems.

INTRODUCTION

In this study the ability to determine the phase speed of a resonant sound signal around a chain of bubbles is demonstrated. The acoustic properties of a single bubble have been known for a long time Rayleigh [1], Minnaert [2]. Minnaert’s equation approximates the frequency of the monopole mode emitted by a single, linearly oscillating bubble under adiabatic condition by

\[ f_0 = \frac{1}{2\pi} \sqrt{\frac{3\gamma P_\infty}{\rho R_0}}, \]

where \( f_0 \) is the frequency in Hz, \( P_\infty \) is the absolute liquid pressure, \( \gamma \) is the ratio of specific heats for the gas, \( \rho \) is the liquid density and \( R_0 \) is the equivalent bubble radius (the radius assuming the bubble is a spherical volume). However, quite different sound spectra and wave speeds develop as multiple bubbles are introduced.

The properties of multiple air bubbles in water have also been broadly studied (Cartensen and Foldy [3], Weston [4], Leighton [5]). Since the passage of sound can be significantly modified by the presence of bubbles and bubble-clusters, the acoustic behavior of such media is important for studies of propagation, reverberation, attenuation and scattering of sound energy Commander et al. [6], Lu et al. [7], Chen et al. [8].

In this paper, a signal processing technique was used to investigate the bubble-scattering effect on the transmission of sound (at frequencies near the resonant frequency) through a chain of uniformly produced bubbles. The smoothed pseudo Wigner-Ville time-frequency technique was employed.

NOMENCLATURE

- \( N \) number of samples
- \( P_\infty \) absolute liquid pressure
- \( R_0 \) equivalent bubble radius
- \( f_0 \) resonant frequency of a single bubble
- \( t \) time variable
- \( \Delta t \) sampling interval
- \( m,n,k \) integer values
- \( x \) input signal
- \( x^* \) complex conjugate of \( x \)
- \( w \) time-frequency distribution
- \( \phi \) arbitrary function – kernel
- \( \gamma \) ratio of specific heats for the gas
- \( \theta \) normalized frequency
- \( \rho \) liquid density
- \( \tau \) time window
- \( \omega \) frequency variable
- \( \Delta \omega \) frequency interval

DESCRIPTION OF THE EXPERIMENTAL SYSTEM

The analysis below was based on time series measurements of the acoustic pressure. The measurements were performed using the experimental set-up illustrated in Figure 1. It is equivalent to the one used by Nikolovska et al. [9] for determining the distribution of the root mean square (RMS) pressure around a bubble chain.
To examine the evolution of the acoustic pulses as bubble number varied from individual to multiple, a small number of bubbles had to be generated. A fixed hydrophone detecting the pulse from the source (the newly-formed bubble) triggered signal acquisition from a traversing hydrophone, ensuring the same pulse initiated signal acquisition in both near and far fields. Pairs of such keyed pulse waveforms from a large number of points in a vertical plane were recorded. No external acoustic forcing was applied.

Figure 1. Experimental set-up

The tank used for the experiments was 0.95 m x 0.56 m in base and 1.5 m deep. The nozzle had a 5.000±0.025 mm internal orifice diameter; it was supplied with air via a precision pressure regulator (CompAir Maxam type A216) at 13.0±0.5 kPa pressure. The nozzle orifice used for bubble production was 1.2 m below the water surface.

A set of two Bruel & Kjaer type 8103 hydrophones with a linear response in the bubble frequency range was used. The hydrophone signals were pre-amplified by Bruel & Kjaer type 2635 charge amplifiers and digitized by a National Instruments Data Acquisition Card type 6024E. The acoustic pressure in the form of voltage output was stored in separate files using StreamTone™ software.

The acoustic center of the first (fixed) hydrophone was at a horizontal distance of 60 mm (point A in Figure 2.) from the nozzle axis (N-S) and in the same vertical level with the nozzle orifice. This position was maintained for all of the experiments while the Bubble-Production Rate (BPR) was varied. The second (scanning) hydrophone was positioned on a 38 × 31 grid (20 mm point to point distance) within the vertical plane (N-S-A) containing the nozzle axis (N-S). The positioning of the second hydrophone was automatically controlled, ensuring an accurate grid positioning required for statistical signal processing.

Two channels, one for the fixed and one for the scanning hydrophone, were logged at 30 kHz each with a 12-bit resolution. The StreamTone™ software running on a Pentium3 class computer was used to acquire data from both channels. The acoustic pressure from both hydrophones in the form of digitized voltage was recorded at every grid-point. The digitized waveforms contained 1024 data points for each bubble pulse. In each file between 36 and 40 bubble pulses were recorded. The software started to record data once a certain voltage trigger level was reached; the signal from the fixed hydrophone was used as the trigger because the signal near the nozzle is highly repeatable Nikolovska et al. [9], Manasseh et al. [10]. Once the triggering occurred, data was recorded on both channels simultaneously. This provided simultaneous acquisition of the sound signal from the source and the far field. Since the hydrophones were different distances from the source there was a small delay time between the two signals owing to the finite speed of sound in water. The maximum possible delay was approximately 0.5 ms.

Figure 2. Hydrophone positioning

TIME-FREQUENCY REPRESENTATION

The time representation of a signal is usually the first and the most natural way of representing a signal because all physical signals are recorded that way. The frequency representation, obtained with the Fourier transform, is also a powerful way to describe a signal mainly because it decomposes the signal into the fundamental harmonics that describe the signal.

However, plots of the Fourier frequency spectrum only contains information regarding the wave frequency that exists in the signal but does not have any information about the time at which these frequencies occur.
Time-frequency distributions provide a technique for investigating how the frequency content of a given signal changes as a function of time. The output of these distributions is the energy density, or intensity, of various components of a signal at given points in time. Cohen [11] has presented a thorough review of time-frequency distributions and discussed their numerical applications. From Cohen [11], the general equation for a time-frequency distribution, \( w(t, \omega) \), for an input signal \( x(t) \) is given by:

\[
w(t, \omega) = \frac{1}{2\pi} \iint e^{-i\phi(\beta, \tau)} x^*(u - \frac{\tau}{2}) x(u + \frac{\tau}{2}) du d\tau d\theta.
\] (2)

where the integrals are evaluated from \(-\infty\) to \(\infty\). In this equation, \(x^*\) is the complex conjugate and \(\phi(\beta, \tau)\) is an arbitrary function (the kernel), and time and frequency are represented by \(t\) and \(\omega\) respectively. For the Wigner-Ville distribution, which is discussed in more details in Wigner [12], Auger [13], Matz and Hlawatsch [14], the kernel function has a value of 1. Upon this substitution, equation (2) reduces to:

\[
w(t, \omega) = \int x^*(t - \frac{\tau}{2}) x(t + \frac{\tau}{2}) e^{-i\omega \tau} d\tau.
\] (3)

For application to digitized or sampled signals, the equation (3) must be modified to a discrete form. This is given by:

\[
w(m\Delta t, k\Delta \omega) = 2N \sum_{m,n} x^*([m+n]\Delta t) x([m-n]\Delta t) e^{-2i\pi nk/2N}.
\] (4)

In equation (4) \(\Delta t\) is the sampling interval and \(\Delta \omega = \pi/(2N\Delta t)\). The discrete Wigner-Ville distribution in this form requires a higher sampling rate than the conventional Discrete Fourier transform (DFT) to avoid aliasing. Furthermore, the Wigner-Ville distribution produces complicated and unexpected results when the signal contains components of more than one frequency. This can result in a ‘noise’ appearance of signal in the Wigner-Ville distribution at frequencies and times that are not really contained in the waveform. This is caused by interference that consists of cross terms from the multiple frequency components and makes the explanation of results rather complicated. According to Auger [13], one approach that can be used to minimize the effect of ‘noise’ in the distribution is the application of smoothing. Convolving a smoothing window with the distribution will minimize the effects of interference terms in the distribution and eliminates the negative values. The application of a smoothing function to the Wigner-Ville distribution results in the SPWV distribution.

**DATA ANALYSIS**

The SPWV technique described above was used for analyzing the signals recorded from the experiments. Analyses were performed for five different bubble production rates (BPR) of 10 Hz, 12 Hz, 14 Hz, 16 Hz and 18 Hz.

The phase speed and pressure modulation of a resonant bubble sound signal passing a chain of uniformly produced air bubbles was examined. The analysis was carried out on an average of the 36 to 40 sound pulses measured in the grid in the domain around the bubble-chain.

Two parameters were computed: the pressure RMS and the resonant frequency delay. The sound pulses shown on Figures 3, 4 and 5 are the average sound pulses that are recorded near the nozzle (point ‘A’; Figure 2) where the bubbles where produced. The signal from this point was used as a ‘triggering’ pulse, and its propagation in the domain around the bubble-chain was observed. The distributions generated by the SPWV are in the form as shown on Figures 6, 7 and 8.

In this paper figures are presented for few points at 18 Hz BPR, although SPWV analysis were done for all BPRs. The SPWV distribution illustrates a strong variation in time and in space for a given BPR, and also for different BPRs. As shown on Figures 18, 19 and 20, a very significant phase speed delay is notable in the vertical direction (up the chain), but the delay is not so great in the horizontal direction (Figures 12, 13 and 14). These distributions were generated from the corresponding pulses shown in figures 9, 10, 11, 15, 16 and 17.

The phase speed can be calculated by taking the time of appearance of the peak of the resonant frequencies together with the known propagation distance (the distance was calculated through the measuring-point coordinates).

As the BPR was increased the number of bubbles in the tank was also increased from 30±4 to 10 Hz to 60±6 for 18 Hz. The results show that the signal energy and the phase speed in vertical direction were significantly affected by the presence of the bubble-chain as well as by the increase of the bubbles population (Table 1.) and hence void fraction. This is in consistency with the results presented by Feuillade [15] for the same bubble sizes (2.59 [mm] to 3.17 [mm]) and corresponding volume fractions of 0.058% to 1% (these volume fractions are calculated for a ‘virtual’ cylinder with the radius equal to the distance to point ‘A’ and with the height of the tank). Feuillade in [16] applied sound to the system. He illustrated that as the volume fraction increases there is a strong ‘coupling’ effect occurring, especially if the bubbles are of the same size and if the observed insonified sound pulse has a frequency near the resonant frequency of individual bubbles.

<table>
<thead>
<tr>
<th>BPR [Hz]</th>
<th>phase speed [m/s] in horizontal</th>
<th>phase speed [m/s] in vertical</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>608 ± 70</td>
<td>275 ± 50</td>
</tr>
<tr>
<td>12</td>
<td>475 ± 65</td>
<td>200 ± 55</td>
</tr>
<tr>
<td>14</td>
<td>280 ± 50</td>
<td>129 ± 60</td>
</tr>
<tr>
<td>16</td>
<td>197 ± 63</td>
<td>102 ± 30</td>
</tr>
<tr>
<td>18</td>
<td>120 ± 45</td>
<td>58 ± 10</td>
</tr>
</tbody>
</table>

Table 1.
Figure 9. Average signal at point 'A' - 60mm in horiz.

Figure 10. Average signal at point 'B' - 120mm in horiz.

Figure 11. Average signal at point 'C' - 300mm in horiz.

Figure 12. SPWV for the signal at point 'A'

Figure 13. SPWV for the signal at point 'B'

Figure 14. SPWV for the signal at point 'C'
Figure 15. Average signal at point 'D' - 120mm in vert.

Figure 16. Average signal at point 'E' - 300mm in vert.

Figure 17. Average signal at point 'F' - 600mm in vert.

Figure 18. SPWV for the signal at point 'D'

Figure 19. SPWV for the signal at point 'E'

Figure 20. SPWV for the signal at point 'F'
Considering the RMS pressure the impact of the higher energy of the resonant frequency on the overall pressure RMS is considerable. Figures 21, 22 and 23 represent plots of the energy of the resonant frequency band (0.7 kHz, 1.0 kHz) of a measured bubble pulses. Effective digital ‘noise’ filtering was performed by use of the SPWV spectrums (this was done for every grid point individually). The anisotropy in the acoustic pressure observed by Nikolovska et al. [9] and Manasseh et al. [10] was also noted here, although the high and low frequency noise was contributing signal’s overall energy.

The graph at Figure 24 summarizes the BPR effects on the overall frequency delay. The blue line represents the mean values of the calculated phase speed in the horizontal direction along with error bars for the corresponding values, and the red line represents the phase speed in vertical direction. There is clearly a decrease in the phase speed for different BPR. As the BPR is increased, the phase speed drops more rapidly and the difference between the phase speed in the horizontal and the phase speed in the vertical direction is decreased.

**CONCLUSIONS**

The case investigated has two main features: firstly, the bubbles are continually produced by a system that ensures a constant bubble size thus providing a series of “natural resonators”; secondly, the sound applied to the chain by bubble formation has the resonant frequency of the bubbles.

The frequencies emitted as uniform spherical bubbles are formed in a chain can be seen to change progressively with time during the acoustic pulse. Furthermore, as bubbles are brought closer together owing to more rapid bubble production, there is a marked drop in the phase speed of the propagation of the resonant frequency.

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