THE EFFECT OF GAS DIFFUSION ON BUBBLE DYNAMICS

B. Brunn/Darmstadt University of Technology
Chair of Turbomachinery and Fluidpower
brunn@tfa.tu-darmstadt.de

G. Ludwig/ Darmstadt University of Technology
Chair of Turbomachinery and Fluidpower
ludwig@tfa.tu-darmstadt.de

B. Stoffel/ Darmstadt University of Technology
Chair of Turbomachinery and Fluidpower
stoffel@tfa.tu-darmstadt.de

ABSTRACT

The progression of cavitation nuclei in a facility determines the cavitation behavior of the devices built in. Thus the cavitation performance of a pump right after a certain downtime is different from that at permanent operation. As it is known also, any variation of system pressure, flow rate or concentration of dissolved gases has an effect on the cavitation nuclei and, therefore, on the tensile strength of the liquid.

To quantify those influences the advanced Rayleigh-Plesset equation has been combined with a diffusion model for the transfer of gases across the wall of a bubble. Thereby, a theoretical model is available to study the growing and shrinking of a spherical bubble in a liquid flow under varying conditions.

Particularly, the shell and the area effect cause a close interaction between pressure driven bubble dynamics and bubble growth by gas diffusion. Any pressure variation initiating oscillation of the bubble affects an increase of the gas content of the bubble. Consequently, turbulence that produces permanent bubble oscillation is of utmost importance.

The numerical model allows a spatio-temporal consideration of the dissolved gas concentration field around the bubble. This provides a further insight into the interaction of bubble dynamics and gas diffusion. Numerical results of bubble size variations are shown under the combined effects of pressure variations and concentration of dissolved gas.

INTRODUCTION

The numerical model introduced below for cavitation nuclei including bubble dynamics and diffusion processes has been developed in the frame of a research project on the development of cavitation nuclei in industrial facilities financially supported by the KSB-Stiftung, Heidelberg (KSB foundation).

As the amount and size of micro bubbles inside the flow is responsible for the local tensile strength of the liquid, they have a high importance for the cavitation performance of any device in the facility. In practice, besides pressure the content of dissolved gas in the liquid primarily determines the bubble size distribution. Thus, at different gas content of the liquid in the facility the cavitation inception of an axial pump can considerably differ [1] (see Figure 1).

In addition to experimental studies, the numerical model of a micro-bubble provides an insight into the physical processes causing growth and shrinkage of the bubbles and the associated variations of the tensile strength.

The thereby achieved theoretical conclusions facilitate the understanding and interpretation of experimental results. At a higher level, the numerical model allows a prediction of the tensile strength allocation in a facility, and the cavitation state of a device in a certain operating condition becomes predictable.

Essential for the influence of the turbulence on cavitation nuclei are the so-called shell and area effects, that are described in detail below. Both effects appear whenever a bubble is oscillating and cause diffusion of gas from the surrounding liquid towards and into the bubble. As turbulence stimulates bubble
oscillation, it initiates this rectified diffusion [2] and causes growing of the bubble in that way. Therefore, turbulence may be a cause for the unexplained long lifetime of cavitation nuclei in the liquid flowing in a facility.

The effective lifetime of micro bubbles is noticeable during the start-up of a pump in a closed loop. In calm water micro-bubbles are dissolved or rise to the surface, so straight after the start-up there are no cavitation nuclei in the flow and cavitation inception in the pump occurs at very low cavitation numbers. If now the pump is cavitating, it generates micro-bubbles that can survive a whole circulation in the loop and act as cavitation nuclei. The consequence is an increased cavitation susceptibility of the pump and thus cavitation inception at higher cavitation numbers.

The numerical model introduced here supports a deeper insight into the correlation of bubble dynamics and gas diffusion. The results shown below concern especially the importance of turbulence and rectified diffusion for the stability of cavitation nuclei. The micro bubble model programmed in the language Matlab Simulink, a product of The Mathworks, Inc., According to the common applications, in the simulations the liquid is water and the gas inside the bubble and dissolved in the surrounding liquid is air.

**NOMENCLATURE**

- \( a \) speed of sound in the liquid \[ \text{m/s} \]
- \( A \) surface area for diffusion \[ \text{m}^2 \]
- \( c \) concentration of dissolved gas \[ \text{kg/m}^3 \]
- \( c_s \) concentration at saturation \[ \text{kg/m}^3 \]
- \( c_x \) in the surrounding liquid \[ \text{kg/m}^3 \]
- \( D \) diffusion constant liquid-gas \[ \text{m}^2/\text{s} \]
- \( f_e \) eigenfrequency of a bubble \[ \text{Hz} \]
- \( H \) Henry coefficient \[ \text{bar}/\text{vol}\% \]
- \( m \) mass of gas inside the bubble \[ \text{kg} \]
- \( n \) polytropic exponent \[-\]
- \( p_g \) partial pressure of gas in the bubble \[ \text{Pa} \]
- \( p_v \) vapour pressure of the liquid \[ \text{Pa} \]
- \( p_s \) static liquid pressure far from bubble \[ \text{Pa} \]
- \( q = \frac{V}{V_n} \) specific volume flow rate \[-\]
- \( r \) polar coordinate \[ \text{m} \]
- \( R \) bubble radius \[ \text{m} \]
- \( R_e \) \( R \) at static equilibrium \[ \text{m} \]
- \( \dot{R} \) velocity of \( R \) \[ \text{m/s} \]
- \( \ddot{R} \) acceleration of \( R \) \[ \text{m/s}^2 \]
- \( R_g \) gas constant \[ \text{J/kgK} \]
- \( t \) time \[ \text{s} \]
- \( T \) temperature \[ \text{K} \]
- \( V \) bubble volume \[ \text{m}^3 \]
- \( \dot{V} \) flow rate \[ \text{m}^3/\text{h} \]
- \( \dot{V}_n \) nominal flow rate \[ \text{m}^3/\text{h} \]
- \( \alpha_s \) concentration at saturation \[ \text{vol}\% \]
- \( \Delta p_{fr} \) pressure offset by friction/viscosity \[ \text{Pa} \]
- \( \Delta p_s = \frac{2 \tau}{R} \) pressure offset by surface tension \[ \text{Pa} \]
- \( \Delta p_v \) \( p_v \) offset by evaporation enthalpy \[ \text{Pa} \]
- \( \rho_n \) nominal density of the gas \[ \text{kg/m}^3 \]
- \( \rho_c \) liquid density \[ \text{kg/m}^3 \]
- \( \sigma \) cavitation number \[-\]
- \( \sigma_i \) \( \sigma \) at incipient cavitation \[-\]
- \( \tau \) surface tension \[ \text{N/m} \]

**BUBBLE DYNAMICS**

The bubble dynamics describe the time-dependent variation of the bubble size. For this purpose, the balance of normal forces acting on the bubble wall is considered. The simplest case is a balance of forces with no velocity and acceleration of the bubble wall. Equation 1 describes this equilibrium.

\[
p_g + p_v = p_s + \Delta p_s \tag{1}
\]

The pressure forces on the left side of Equation 1 are acting outwards, the right ones are directed to the center of the bubble. Imbalance of these forces leads to acceleration of the bubble wall and variation of the bubble size. Then by inertia and viscous drag additional forces arise. The complete bubble dynamics are described by the advanced Rayleigh-Plesset-Zwick equation (see e.g.[3]):

\[
\begin{align*}
\left(1 - \frac{\ddot{R}}{a}\right) R \cdot \ddot{R} + \left(1 - \frac{\dot{R}}{3a}\right) \frac{3}{2} \dddot{R} & = \\
\left[1 + \frac{\dot{R}}{a}\right] \frac{3}{4} \frac{\Delta p}{p_s} + \left[1 - \frac{\dot{R}}{a}\right] \frac{1}{\rho_c} \frac{\partial p}{\partial t} \\
\end{align*}
\tag{2}
\]

with

\[
\Delta p = p_v (\vartheta) + \Delta p_v + p_g - \Delta p_s - \Delta p_{fr} - p_s \tag{3}
\]

The usual motion of the bubble wall is an oscillation around a mean bubble radius that is mainly defined by the equilibrium of the wall forces given in Equation 1. At given liquid properties and pressure, the equilibrium radius is only dependent on the partial pressure \( p_g \) of the gas inside the bubble. The partial pressure results from the ideal gas law:

\[
p_g = \frac{m \cdot R_g \cdot T}{V} = \frac{4}{3} \pi R^3 \tag{4}
\]

Consequently, partial pressure and thus equilibrium radius are determined by the mass of gas \( m \) inside the bubble. Figure 2 shows the response of bubbles with different gas masses on a sudden pressure drop from 2bar to 0.5bar without diffusion. The free oscillation of the bubbles induced by the pressure drop also shows the effect of the gas mass inside the bubble on the bubble eigenfrequency.
As the mass of gas inside the bubble is changed by gas diffusion, diffusion processes around the bubble are most important for the mean bubble size and the bubble dynamics. The time dependent progress of the gas diffusion determines the growing or shrinking rate of a bubble and, thereby, its stability.

DIFFUSION MODEL

The assignment of the diffusion model is the computation of the time dependent variation of the gas mass inside the bubble. As the transport of gas across the bubble wall occurs without any delay, the velocity of gas transport (gas diffusion) in the surrounding liquid is decisive for the exchange of gas between liquid and bubble. The mass transport by diffusion is given by:

\[ \dot{m} = A \cdot D \cdot \frac{\partial c}{\partial r} \]  

Thus, most important is the radial gradient of the gas concentration in the liquid surrounding the bubble. To calculate this gradient over the time, the concentration field around the bubble has been discretised as shown in Figure 3.

![Figure 3: Discretization of the Concentration Field](image)

The numerical diffusion model generates 50 layers around the bubble with fixed volumes, so that for each layer a gas concentration can be calculated from the particular mass of dissolved gas. So it is possible to compute and analyze the time dependent concentration field in the surrounding liquid. This helps to understand the correlation of bubble dynamics and diffusion.

At each time step, the gas exchange between the neighbouring layers is computed with the instantaneous concentration gradients at each boundary of the discretized layers (see Equation 5) and the resulting new gas concentrations of the layers are recorded.

The discretisation is fine enough, respectively the number of layers is sufficient, if the resulting concentration field shows continuous gradients at any time. The radius of the outer layer marks the size of the modelled concentration field. If the concentration of the outer layer keeps almost equal to the concentration in the water beyond and the concentration gradient goes outward to zero, the modelled concentration field is also large enough.

As shown in Equation 5, gas diffusion across the liquid is caused by different gas concentrations. The highest gradients can be found close to the bubble, because the gas concentration at the bubble wall, according to Henry’s law, is always the saturation concentration:

\[ c_s = \rho_n \cdot a_s = \frac{\rho_n \cdot P_g}{100\% \cdot H} \]  

Therefore, the concentration at the bubble wall is determined by the partial pressure of the gas inside the bubble. Any variations of the partial pressure result in a corresponding concentration gradient at the bubble wall. Thereby, gas diffusion across the bubble wall is initiated (see Equation 5) and gas content as well as partial pressure of the bubble are changing. This leads to a close interaction between partial pressure, concentration gradient and gas transport. According to the previous equations, the partial pressure is the link between dynamics and diffusion.

![Figure 4: Exemplary Concentration Distribution](image)
concentration distribution in the liquid is important insofar as it affects the gas diffusion and thus the gradient at the wall in the further progress.

**SHELL AND AREA EFFECTS**

An oscillating bubble passes through alternating phases of expansion and compression. During expansion the partial pressure in the bubble and, thereby, the saturation concentration (see Equation 6) is decreasing. Thereby, a positive concentration gradient is generated as shown in Figure 4, and gas diffuses into the bubble. The inverse effect is caused by the compression that leads to temporary diffusion of gas out of the bubble into the surrounding liquid.

Because of two rectifying effects the alternating diffusion of gas into and out of the bubble is not balanced. First variations of the bubble size cause different thicknesses of the layers, whose volumes keep constant (see Figure 3). Although the gas concentrations of the layers remain the same, the concentration gradient is varying by the deformation of the layers. So, during expansion and diffusion of gas into the enlarged bubble the concentration gradient and, thereby, the intensity of the diffusion (see Equation 5) is increased.

Secondly, a widened bubble leads to expanded layers with larger surface areas. The surface areas of the layers are the effective surface areas for the diffusion. According to Equation 5, the gas diffusion into the enlarged bubble during expansion is increased again. So, both effects lead to an intensified diffusion of gas into the bubble in the phases of expansion. Thus, an oscillating bubble causes rectified diffusion toward the bubble wall, and the mean bubble size bubble is growing with time.

The numerical model allows the mathematical elimination of the shell effect as well as of the area effect. So, the magnitude of both effects can be isolated and it can be made sure that no other factors affect the diffusion processes. A pressure oscillation with gradually increasing amplitudes provides a balanced diffusion with almost no variation of the gas mass if shell and area effect are eliminated. Figure 5 displays the gas mass of a bubble that is exposed to a pressure oscillation with an amplitude of 0.5 bar and a frequency of 1 kHz. Both effects are separately and collectively eliminated or activated, respectively. The simulation results show that the influences of both effects are almost equal. Equation 5 already indicates that the concentration gradient and the surface areas have the same importance for the mass transport by diffusion.

The higher time gradient of the gas mass inside the bubble at the beginning is caused by the initially uniform concentration field in the surrounding liquid. The instantaneous gas mass near the bubble wall can be transported across the bubble wall comparatively fast. Further amounts of gas diffusing towards the bubble have to overcome increasing distances. Thereby, bubble growth slows down more and more. But if pressure and gas concentration in the surrounding liquid don’t change, the increase of the bubble does not end until the buoyancy force becomes too high and the bubble rises to the liquid surface.

**Figure 5: Gas Mass of an Oscillating Bubble**

The progress with time of the concentration field in the surrounding liquid of the bubble shown in Figure 5 (both effects active) is displayed in Figure 6. One can see the concentration distribution at several time steps, each at the end of a compression (black) or an expansion phase (gray). The concentration near the bubble is very dynamic because of the alternating saturation concentration. However, the average concentration near the bubble wall is beyond the initial concentration. The missing amount of gas already diffused into the bubble and causes the initial high gradient of the gas mass inside the bubble found in Figure 5.

At a distance of 2-4 µm from the bubble wall, the concentration keeps permanently below the concentration of the outer liquid. The consequence is an always positive concentration gradient (in outward direction) and, therefore, a permanent transport of dissolved gas from the outer liquid towards the bubble. If the bubble and the discretized diffusion layers are regarded as closed system the amount of gas within this system is thereby continuously increasing. As the average concentrations of the layers rather decrease, the additional amount of gas diffusing into the system are transported into the bubble and cause the permanent bubble growth seen in Figure 5.

**EFFECT OF TURBULENCE**

Basically, there is no stable equilibrium for the diffusion process between a bubble and the surrounding liquid. With ongoing time bubbles either shrink and dissolve or grow and rise
to the surface. Therefore, as it is well known, after sufficient
time calm water doesn’t contain any free gas bubbles. On the
other hand, whenever the water is moved and pressure fluctua-
tions are generated, new free bubbles are produced within the li-
quid and can grow fast.

Pressure fluctuations as they are caused by turbulence in-
duce continuous bubble oscillation and, thereby, the described
rectified diffusion that increases the gas mass inside the bubble.
Thus, turbulence may be a factor that contributes to the still un-
explained stability of cavitation nuclei in industrial facilities.

To understand the effect of turbulence, it is important to
know the influence of frequency and amplitude of the pressure
fluctuation on the intensity of the gas diffusion. It is obvious
that a higher amplitude of the pressure fluctuation leads to a hig-
her amplitude of the oscillating bubble radius and thereby intens-
sifies the shell and area effect that are generally induced by vari-
ations of the bubble size. Figure 7 shows the influence of the
amplitude of the pressure oscillation on the gas diffusion at a
frequency of 1 kHz.

Figure 7: Influence of Pressure Amplitude on Rectified
Diffusion

The influence of the pressure frequency is much more com-
licated as a gas bubble is a complex vibratory system with higher
harmonics. The gas content of the bubble acts as gas spring,
the inertia is provided by the mass of the surrounding liquid that
has to be accelerated or decelerated when the bubble size varies,
and the system is damped by the viscosity and the compress-
sibility of the liquid. Apart from the forced resonance vibrations
of the bubble that lead to different amplitudes of the bubble ra-
dius oscillations there is no influence of the pressure frequency
itself, presumed that time span and range of the compression
and expansion phases are equal. It makes no difference if in a
certain time duration there are few long compression and expan-
sion phases or many short ones. Of importance is only the accu-
mulated duration of the respective phases.

Figure 8 displays the amplitude spectrum of a bubble based
on the frequency of the liquid pressure oscillation and also the
respective increase of the gas mass inside the bubble by rectified
diffusion within a time of 1 ms. In this example, a bubble of 20
µm radius is exposed to a pressure oscillation with an offset of
0.78 bar and an amplitude of 0.1 bar. Obviously the influence of
the pressure frequency on the gas diffusion is mainly caused by
the different amplitudes of the bubble radius.

Figure 8: Influence of Pressure Frequency on Rectified
Diffusion

If there is no influence of the pressure frequency itself, two
different frequencies with equal amplitudes of the bubble radius
should affect the same gas diffusion and, thereby, the same in-
crease of the gas mass inside the bubble. To verify this consi-
deration, the pressure frequencies 18.68kHz and 173.29kHz had
been chosen which cause the same amplitude of the bubble ra-
dius of 0.860µm. Figure 9 shows the comparison between the
progress with time of the respective gas masses inside the bubble.

Figure 9: Isolated Influence of Pressure Frequency

The results document that in the case of the lower frequen-
cy the diffusion is more intensive although the radius amplitudes
are equal. The reason is an additional influence of the pressure
frequency on the mean radius of the oscillating bubble. Figure
10 shows the progress with time of the respective radii. The am-
plitudes are equal but the mean radius caused by the lower pres-
sure frequency is larger than the equilibrium radius of 20µm.
Thereby, the bubble has stronger expansion phases and the dif-
fusion of gas into the bubble is increased.
The general influence of the pressure frequency on the mean bubble radius is shown in Figure 11. The results confirm a mean radius of 20.08µm at the frequency of 18.68kHz and a mean radius equal to the equilibrium radius of 20µm for the frequency of 173.29kHz.

Although there is an additional influence of the mean bubble radius on the gas diffusion, the effect of the radius amplitude is dominant. The highest amplitude of radius of the bubble shown in Figure 8 (equilibrium radius: 20µm, mean pressure: 0.78 bar) and, thereby, the highest increase of gas mass appears at a frequency of 115.6 kHz. This is not the eigenfrequency of the bubble, and the resulting amplitude is the sum of the fundamental oscillation and several harmonics.

The real eigenfrequency can be found by a shock induced free oscillation, as usual. After a pressure shock the bubble of 20µm at the mean pressure of 0.78bar oscillates with its eigenfrequency of 123.57kHz. If friction, compressibility of the liquid and thermodynamic effects are neglected the eigenfrequency of a bubble can also be computed by Equation 7 [3]. In the current case, Equation 7 gives an eigenfrequency of 123.55kHz that agrees well with the result from the simulation of free bubble oscillation.

\[
f_c = \frac{1}{2\pi R_c} \sqrt{\frac{3n \cdot (p_\infty - p_v) + (3n - 1) \cdot \frac{2 \cdot \tau}{R_c}}{p_\infty}}
\]  

Figure 12 shows the Fast Fourier Analysis of the bubble response on two different pressure frequencies. The gray graph shows the bubble response with the maximum first-order response (response frequency = excitation frequency). After the rules of an order analysis this is the characteristic of the eigenfrequency. Thus, also the simulation of a forced vibration verifies the eigenfrequency of 123.6 kHz.

Nevertheless, in terms of gas diffusion not the eigenfrequency is decisive but the resonance frequency which produces the highest total amplitude. Figure 12 also displays the Fourier Analysis of the response to the respective pressure frequency of 115.6 kHz. The first-order response is by definition lower than in case of the eigenfrequency, but all harmonics together lead to a maximum total amplitude. The results presented in of Figure 8 confirm that this pressure frequency causes the highest gas diffusion.

**CONCLUSIONS**

The concentration field around a micro-bubble has been modelled to simulate diffusion processes that cause long range growing and shrinking of a cavitation nucleus within the flow. Thereby, the enhanced Rayleigh model for bubble dynamics has been extended by the effect of gas diffusion.

The variation of the gas mass inside the bubble by diffusion affects the partial pressure and the equilibrium radius and, as a consequence, the mean size of the bubble. On the other hand, the partial pressure is dependend on the instantaneous bubble radius and determines the saturation concentration at the bubble wall, which induces the diffusion processes.

Consequently, there is a close interaction between bubble dynamics and gas diffusion. The gained insight into the concentration field helps to better understand the long term behavior of the size of micro-bubbles, which is decisive for the local tensile
strength of liquids and the cavitation inception in flow devices and turbomachines.

Rectified diffusion is an important effect in respect to diffusion processes and to the effects of pressure surges and turbulence on bubble lifetime. The possibility to eliminate the shell and/or the area effect in the numerical simulation provides information on the effective influence of the rectified diffusion and the contribution of the respective effects. For the mass transport by gas diffusion the radius amplitude of the oscillating bubble is decisive. Concerning the frequency dependance, the resulting radius amplitude is most important. The highest increase of gas mass within the bubble is affected by the resonance frequency with the highest radius amplitudes, that differs from the eigenfrequency. The variation of the mean bubble radius caused by the pressure frequency is of comparatively minor influence.

Within the current project an additional verifying of the numerical results by experimental investigations is planed also.

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