Velocity Field and Pressure Distribution Around Two Parts of a Cavitation Bubble after its Splitting near a Rigid Boundary

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Abstract
Previous studies on the behaviour of the two parts of a cavitation bubble after its splitting showed that, under certain conditions, while the upper part of the bubble remains almost spherical, the lower part of the bubble migrates towards the rigid boundary with a liquid jet which is developed on the far side of the bubble from the rigid boundary and is directed towards it. The impingement of the liquid jet towards the rigid boundary causes the cavitation damage. In this paper a Boundary Integral Equation Method is employed to simulate the dynamics of the two parts of the cavitation bubble after its splitting. The velocity field and the pressure distribution around the two parts of the cavitation bubble after its splitting is obtained by coupling the Boundary Integral Equation Method and the Finite Difference Method. The velocity field illustrates the liquid flow around the two parts of the cavitation bubble after its splitting and indicates the existence of a stagnation point between the two parts of the bubble. The pressure distribution around the two parts of the bubble after its splitting illustrates the point of maximum pressure between the two parts of the bubble. It is shown that the stagnation point and the point of maximum pressure does not coincide as it is expected in steady problems. This is because of the unsteady nature of the problem under investigation.

Introduction
Experimental and numerical investigations on the behaviour of a cavitation bubble near a rigid boundary have shown that the cavitation bubble in the absence of buoyancy forces, during its collapse phase, is attracted by the rigid boundary [1-3]. Experimental and numerical studies have also shown that a buoyant vapour bubble in the vicinity of a rigid boundary has different behaviors different circumstances [4]. In the case of weak buoyancy forces, the bubble migrates towards the rigid boundary with a high speed liquid jet directed towards the boundary. In the case of strong buoyancy forces, the bubble migrates upwards away from the rigid boundary.
surface with a liquid jet directed away from the surface. Our previous studies [5] on the dynamics of a cavitation bubble above a rigid boundary have shown that in the case that neither the buoyancy forces nor the Bjerknes attraction forces through the rigid boundary dominates the physics of the problem, the bubble during its collapse phase takes the shape of an hour-glass and splits into two parts. In this paper velocity field and pressure distribution after splitting of the bubble into two parts is calculated by using the Boundary Integral Equation Method and the Finite Difference Method.

Cavitation bubble at the instant of splitting
During collapse phase of the cavitation bubble above a rigid surface, an annular liquid jet is developed around the bubble and the bubble is transformed into shape of an hour-glass. This is necking phenomenon which is followed by splitting of the bubble into two parts. The initial distribution of the velocity potential on the bubble surface and on the rigid boundary is at the instant of splitting and is obtained from the last stage of the necking phenomenon. The pressure inside the upper and lower parts of the bubble at the instant of splitting is the same and is equal to the pressure inside the bubble at the last stage of the bubble necking. The pressure inside the bubble is made up of (by) partial pressures of a constant vapour pressure and a variables pressure of non-condensable gases. It is assumed that non-condensable gases behave as an ideal gas.

Green’s formula for potential flows
The liquid domain around the two parts of the cavitation bubble is assumed to be incompressible, inviscid and irrotational and the surface tension is assumed to be negligible. Therefore the Green’s formula for potential flows can be applied for simulation of the liquid flow around the two parts of the bubble after its splitting

\[
C(p)\Phi(p) + \int_S \Phi(q) \frac{\partial}{\partial n} \left( \frac{1}{|p-q|} \right) ds = \int_S \frac{\partial}{\partial n} \left( \Phi(q) \right) \left( \frac{1}{|p-q|} \right) ds
\]

Where \( q \) is any point on the surface of the bubbles and on the rigid boundary, \( p \) is any point on the liquid domain; \( s \) is the surface of the bubbles and the surface of the rigid boundary. While \( C(p) \) is equal to \( 2\pi \), when \( p \) is on the boundary and is \( 4\pi \) when \( p \) belongs to the liquid domain.

The Lagrangian form of the unsteady Bernoulli equation is employed to obtain the velocity potential on the surface of the two separated bubbles in successive time steps

\[
\frac{D\phi}{Dt} = \frac{P_\infty - P_v}{\rho} - \frac{1}{2} \left| \nabla \phi \right|^2 + g(z-h)
\]

Where \( P_\infty \) is pressure in the infinity and \( P_v \) is pressure inside the bubble.

Discretization of the boundaries
The boundaries of the upper and lower parts of the bubble after its splitting is discretized by MU and ML linear segments and the rigid surface is discretized by MR linear elements. The velocity potential and the normal derivative of the velocity potential is constant along each elements and are located on the mid point of each element. The discretized form of the Green’s formula is given by
\[2\pi \Phi(P_i) + \sum_{b=i}^M \left( \Phi(q_j) \int \frac{\partial}{\partial n} \left( \frac{I}{p_i - q_j} \right) dS \right) = \]

\[\sum_{j=1}^M \left( \frac{\partial}{\partial n} \left( \Phi(q_j) \right) \left( \frac{I}{p_i - q_j} \right) dS \right) \quad (3)\]

A variable time step is employed as

\[\Delta t = \min \left( \frac{\Delta \Phi}{\frac{P_e - P_f}{\frac{1}{2}VI^2 + \sigma^2(z - y)}} \right) \quad (4)\]

Where \(\Delta \Phi\) is some constant. The Runge-Kutta integration scheme of the second order is used for evolution of the upper and lower parts of the bubbles in successive time steps.

**Discretization of the liquid domain**

The liquid domain is discretized by even spaced fixed points. The Green’s formula is used for obtaining the velocity potential on each fixed point inside the liquid domain and on its four neighbor points (see figure 1).

Figure 1

By having the velocity potential on the four neighbor point of each fixed point inside the domain and by employing a finite difference scheme, the vertical and radial components of the velocity on each fixed points can be obtained. By having the velocity potential on the fixed points in the two successive time steps and by using the Eulerian form of the unsteady Bernoulli equation, the pressure of the liquid domain on each fixed point is calculated.

**Numerical results and discussion**

Figure 2 illustrates the shape of the upper and lower parts of the cavitation bubble above a rigid boundary after its splitting into two parts at the non-dimensional time \(t = 2.63605\). This figure also illustrates the velocity field in the liquid domain around two parts of the bubble. In this figure it is observed that a stagnation point exists between two parts of the bubble and on the axis of symmetry.

Figure 2 indicates that while the upper bubble takes the shape of rugby ball, the side of the lower part of the bubble far from the rigid boundary is flattened and a liquid jet is initiated on that side of the bubble and is directed to the rigid surface. Figure 3 shows the pressure contours in the liquid domain around the bubbles. It is shown that a point of maximum pressure is located above the lower part of the bubble and on the axis of symmetry. Figures 2 and 3 indicate that the stagnation point and the point of maximum pressure are not coincide. This phenomenon is because of the unsteady nature of the problem.

Figure 4 illustrates the shape of the two parts of the bubble after its splitting and the velocity field around the bubbles at the non-dimensional time \(t = 2.70762\). Figure 4 indicates that while the upper bubble remains almost in the shape of a rugby ball, the liquid jet on the far side of the lower bubble from the rigid boundary is developed and is directed to the rigid surface. Figure 5 shows the pressure contours around the bubbles. It is shown that the point of maximum pressure is located on the axis of symmetry.
symmetry and inside the liquid jet. Figures 4 and 5 show that the stagnation point is located above the point of maximum pressure. Figure 6 illustrates the shape of the upper and lower parts of the bubble and the velocity field around the bubble after its splitting above a rigid boundary at the non-dimensional time $t = 2.75890$. Figure 6 illustrates that while the upper parts of the bubble still remain in the shape of a rugby ball, the liquid jet, which is developed on the far side of the lower bubble from the rigid boundary, penetrates the opposite side of the bubble boundary and transforms the lower bubble into a toroidal bubble. Figure 7 shows the pressure contours in the liquid domain around the bubble at the non-dimensional time $t = 2.75890$. It is shown that the point of maximum pressure is located inside the liquid jet and on the axis of symmetry.

**Concluding remarks**

Previous studies on the dynamics of a cavitation bubble above a rigid boundary have shown that during the collapse phase of the bubble, the point of maximum pressure is located on the axis of symmetry and above the liquid jet [1]. Our results in this paper indicate that in the case of the bubble splitting the point of maximum pressure is located inside the liquid jet which is developed on the far side of the lower bubble and is directed to the rigid surface.
Figure 6

Figure 7
Reference