STEADY-STATE CAVITATING NOZZLE FLOWS WITH NUCLEATION

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ABSTRACT
Quasi-one-dimensional steady-state cavitating nozzle flows with bubble nucleation and nonlinear bubble dynamics are considered using a continuum bubbly liquid flow model. The onset of cavitation is modeled using an improved version of the classical nucleation equation and the nonlinear dynamics of cavitating bubbles is described by the classical Rayleigh-Plesset equation. Using a polytropic law for the partial gas pressure within the bubble, stable steady-state flows with stationary shock waves as well as unstable flashing flow solutions were observed, similar to the homogeneous nozzle flow solutions given by Wang and Brennen[1] and by Delale et al.[2]. In particular, the unstable flashing flow instabilities encountered can be stabilized by accounting for thermal damping and/or flow unsteadiness.

INTRODUCTION
Recent investigations of steady-state cavitating nozzle flows of homogeneous bubbly mixtures using spherical bubble dynamics with a polytropic law for the partial gas pressure (e.g. see Wang and Brennen[1] and Delale et al.[2] ) have shown flow instabilities which correspond to unphysical solutions. The existence of such instabilities has suggested the need for improvements and modifications in the original model equations. By taking thermal damping into account in a model similar to that given by Prosperetti[3], Delale[4] has shown that the steady-state instabilities encountered in homogeneous

NOMENCLATURE

\( A' \) : the cross-sectional area of the nozzle
\( C_p \) : the pressure coefficient
\( Gb \) : Gibbs number
\( H' \) : Henry’s constant
\( H'_i \) : inlet nozzle height
\( J' \) : the nucleation rate
\( M \) : flow Mach number
\( R' \) : bubble radius
\( R'_i \) : critical bubble radius at the onset of cavitation
\( Re \) : flow Reynolds number
\( S' \) : the surface tension coefficient
\( T'_L \) : the liquid temperature
\( U'_i \) : inlet flow speed
\( We \) : Weber number
\( Z_v \) : compressibility of vapor
\( c' \) : concentration of dissolved gas in liquid
\( c'_L \) : the speed of sound in liquid
\( k \) : polytropic exponent
\( k_s \) : Boltzmann’s constant
\( m'_i \) : the mass of a single vapor molecule
\( p' \) : mixture pressure
\( p'_b \) : total bubble pressure
\( p'_g \) : partial gas pressure within the bubble
\( p'_l \) : the liquid pressure at the onset of cavitation
\( p'_v \) : partial vapor pressure within the bubble
\( t' \) : the time coordinate
\( u' \) : flow speed
\( x' \) : axial coordinate of the nozzle
\( \alpha \) : correction factor for nucleation work of formation
\( \beta \) : void fraction
\( \delta \) : non-dimensional gas parameter at the onset of cavitation
\( \mu'_L \) : liquid viscosity
\( \nu'_L \) : kinematic viscosity of liquid
\( \ell \) : micro to macro scale
\( \rho_c \) : measure of liquid compressibility
\( \rho' \) : mixture density
\( \rho'_L \) : liquid density
\( \sigma_i \) : inlet cavitation number
bubbly cavitating nozzle flows can disappear to some extent. A recent numerical investigation of unsteady nozzle flows, based on a model similar to that of Wang and Brennen, by Preston et al.[5] shows the possibility that the instabilities in steady-state nozzle flow solutions of homogeneous bubbly mixtures may correspond to bubbly shock waves found in the diverging section of the nozzle and propagated downstream.

The aim of this investigation is to construct and test a cavitating flow model with nucleation. For this reason the model equations for quasi-one-dimensional cavitating nozzle flows with spherical bubble dynamics are modified to take into account the effect of bubble nucleation. An improved version of the classical rate equation for homogeneous bubble nucleation by Delale et al.[6] is applied to determine the onset of cavitation using a parametric investigation, which treats some of the unknown quantities, like the initial partial pressure of the gas within the bubble and the initial radius at the onset of cavitation, as parameters. The nonlinear dynamics is described by the classical Rayleigh-Plesset equation. In particular, steady-state cavitating nozzle flows with nucleation are considered in order to be able to compare the results under similar conditions with those obtained by Wang and Brennen[1] for homogeneous bubbly mixtures. Using a polytropic law for the partial gas pressure, quasi-statically stable solutions with or without stationary bubbly shock waves as well as unstable ‘flushing flow’ solutions were obtained. The unstable solutions can be stabilized by taking into account thermal damping and/or flow unsteadiness.

**A CAVITATING FLOW MODEL WITH NUCLEATION**

In this section we incorporate bubble nucleation into the flow equations and introduce a cavitating flow model with cavitation onset based on a two-phase homogeneous flow description where the relative motion between the bubbles and the surrounding liquid is neglected. A more realistic, but more sophisticated model would require the use of two-fluid equations, one for the gaseous and the other for the liquid phase, together with the interface interactions between the phases. In what follows we introduce a quasi-one-dimensional cavitating nozzle flow model with nucleation. The equations of motion of such flows can be written as

\[ A^2 \frac{\partial \rho'}{\partial t'} + \frac{\partial}{\partial x'} (\rho' u' A') = 0 \quad \text{(1)} \]

\[ \frac{\partial u'}{\partial t'} + \frac{1}{A'} \frac{\partial}{\partial x'} (u' u' A') = J' \quad \text{(2)} \]

\[ \rho' \frac{du'}{dt'} = - \frac{\partial p'}{\partial x'} \quad \text{(3)} \]

where \( \frac{d}{dt'} = \frac{d}{dt} + \frac{u' c'_{l}}{\rho'_{l}} \) is the material or total derivative. The mixture density \( \rho' \) is related to a reference liquid density \( \rho'_{l} \) (say at some reference pressure \( p'_{0} \)) by

\[ \rho' = \rho'_{l} (1 + \rho_{c})(1 - \beta) \quad \text{(4)} \]

where \( \beta \) is the void fraction and \( \rho_{c} \). (1 is introduced to take into account the compressibility of the liquid. For Tait’s equation of state of the liquid, it assumes the form

\[ 1 + \rho_{c} = \left( \frac{p + B'/p'_{0}}{1 + B'/p'_{0}} \right)^{1/m} \quad \text{(5)} \]

where \( p = p'/p'_{0} \) is the normalized pressure and \( B' \) and \( m \) are constants (for water \( B' = 3010 \) atm. and \( m = 7.15 \) for water). If we further assume that the bubbles are spherical and monodispersed with radius \( R' \), the void fraction \( \beta \) can be related to the number density \( n' \) by the relation

\[ \beta = \frac{4}{3} \pi R'^3 n' \quad \text{(6)} \]

Assuming that the gas phase is sufficiently dilute, the nonlinear bubble dynamics can be described by the classical Rayleigh-Plesset equation as

\[ R'^2 \frac{d^2 R'}{dt'^2} + \frac{3}{2} \left( \frac{1}{R'} \frac{dR'}{dt'} \right)^2 = \frac{p'_{s} - p'}{\rho'} - 4 \nu' \left( \frac{dR'}{dt'} \right)^2 - \frac{2S'}{\rho'_{l} R'} \quad \text{(7)} \]

where the total pressure \( p'_{s} \) can be written as the sum of the partial vapor pressure \( p'_{g} \) and the partial gas pressure as

\[ p'_{s} = p'_{g} + p'_{s} \quad \text{(8)} \]

Equations (1)-(8) yield a complete system of model equations for quasi-one-dimensional cavitating nozzle flows, provided that the local nucleation rate \( J' \) and the partial gas pressure \( p'_{s} \) can be evaluated by some reliable means. Such an evaluation of the activated cavitation nuclei by an improved version of the classical theory of homogeneous bubble nucleation will be considered in detail in the next section. A polytropic law for the partial gas pressure will be used here, solely, for comparison reasons with the work of Wang and Brennen[1] despite the fact that thermal conduction and diffusion through the bubble should be taken into account. We can further eliminate the density \( \rho' \) and the number density \( n' \) between eqs. (1), (2), (4) and (6) to arrive at

\[ \frac{d\beta}{dt'} + (1 - \beta) \left[ \frac{\partial u'}{\partial x'} + u' \left( \frac{1}{A'} \frac{dA'}{dx'} \right) \right] + \frac{1}{\rho'_{l} c'_{l}^2 (1 + \rho_{c})} \frac{dp'}{dt'} = 0 \quad \text{(9)} \]

and

\[ \frac{\partial u'}{\partial x'} + u' \left( \frac{1}{A'} \frac{dA'}{dx'} \right) + \frac{1}{\rho'_{l} c'_{l}^2 (1 + \rho_{c})} \frac{dp'}{dt'} - \frac{3 \beta}{R' \frac{dR'}{dt'}} \frac{dR'}{dt'} - \frac{4}{3} \pi R'^3 J' = 0 \quad \text{(10)} \]

where \( c'_{l} \) is the speed of sound in the liquid. Introducing the normalization

\[ u = \frac{u'}{U'_{i}}, \quad \rho = \frac{\rho'}{\rho'_{l}}, \quad p = \frac{p'}{\rho'_{l} U'_{i}^2}, \quad J = \frac{H'^{n}_{i} J'}{U'_{i}} \]
\[ p_b = \frac{p_{b^*}}{\rho_i U_i^{2}}, \quad p_r = \frac{p_{r^*}}{\rho_i U_i^{2}}, \quad p_c = \frac{p_{c^*}}{\rho_i U_i^{2}}, \quad R = \frac{R^*}{R_i}, \]
\[ x = \frac{x'}{H'}, \quad t = \frac{t' U_i}{H'}, \quad A = \frac{A'}{A_i}, \quad t = \frac{R^*}{H'}, \]
(11)

The reduced equations for quasi-one-dimensional nozzle flows with nucleation assume the form
\[ \frac{d\beta}{dt} + (1 - \beta) \left[ \frac{\hat{c} u}{\hat{c} x} + u \left( \frac{1}{A} \frac{dA}{dx} \right) \right] = 0, \]
(12)
\[ \frac{\hat{c} u}{\hat{c} x} + u \left( \frac{1}{A} \frac{dA}{dx} \right) = 0, \]
(13)
\[ R \frac{d^2 R}{dt^2} + \frac{3}{2} \left( \frac{dR}{dt} \right)^2 = \frac{1}{\ell^2} \left[ p_{b^*} - p - \frac{4}{(Re)R} \left( \frac{dR}{dt} \right) - \frac{2}{(We)R} \right], \]
(15)
where \( d/dt = \partial/d\hat{c} + u/\hat{c} \partial \) is the normalized material derivative and the flow Mach number \( M \), the flow Reynolds number \( Re \) and the Weber number \( We \) are defined by
\[ M = \frac{U_i}{c_L}, \quad Re = \frac{H_i U_i}{\nu_L}, \quad We = \frac{p_{L^2}}{\rho_i S^2}. \]
(16)

Equations (12)-(15) with appropriate initial values and nozzle inlet and exit conditions constitute the model equations for quasi-one-dimensional nozzle flows provided that they are supplemented by expressions for the normalized total pressure \( p_{b^*} \) and normalized nucleation rate \( J' \).

**HOMOGENEOUS BUBBLE NUCLEATION AND ONSET OF CAVITATION**

In this section we consider homogeneous bubble nucleation in pure liquids and its extension to gas liquid solutions, in particular, to the problem of cavitation onset. Homogeneous bubble nucleation has been investigated for years both experimentally and theoretically (e.g. see Blander and Katz[7] and references therein). Although nonclassical methods such as the density functional method (Oxtoby and Evans[8]) and molecular dynamics simulations (Kinjo and Matsumoto[9]) can yield a better understanding of the phenomenon, they seem to be of limited practical use in bubbly and cavitating flows. On the other hand, the classical theory can be modified for implementation in such flows because of its simplicity. The classical theory has been criticized for its failure to predict the tensile strengths of liquids at low temperatures and for providing very low steady-state nucleation rates. In a recent paper Delale et al. [6] have corrected for the critical radius and steady-state nucleation rate of the classical theory by constructing a phenomenological nucleation barrier suitable for direct comparison with the results of the experiments. The phenomenological construction of the nucleation barrier in their work is achieved by introducing a phenomenological correction to the free energy of formation of a bubble of critical size. Their results for the critical radius \( r_c' \) and steady-state nucleation rate \( J' \) are given by the expressions
\[ r_c' = \frac{2S'}{p_{r^*} - p_{c^*}}, \]
(17)
and
\[ J' = Z \left( \frac{2S' \rho_i c_H}{\alpha m^3} \right)^{1/2} \left( \frac{4\pi \alpha c_H}{3k_B T_L}(1 - 2\alpha) \right) \]
(18)
where the subscript ex in eq. (17) denotes the value of superheat achieved in experiments rather than the value that would be attained in a reversible process. The correction factor \( \alpha (0 \leq \alpha < 1/2) \) in eq. (18) is introduced to account for the difference between superheat threshold and tensile strength achieved in experiments and that would occur in a reversible process, and is both substance and temperature dependent. In particular, \( \alpha = 0 \) yields the well-known classical expression without correction. Delale et al.[6] show that reasonable nucleation rates can be achieved by setting \( \alpha = 1/3 \) (with the exception of water) for reduced superheat temperatures greater than 0.85 in homogeneous boiling experiments for a variety of substances.

In order to be able to apply the recent modification of the classical theory to the onset of cavitation, we first extend the domain of application of the theory to gas-liquid solutions (mainly, to a liquid with a dissolved gas, containing also particles or Harvey nuclei which can act as cavitation nucleation sites). It has been reported (e.g. see Ward, Balakrishnan and Hooper[10]) that in weak solutions the critical radius is given by the equilibrium radius in the form
\[ r_c' = \frac{2S'}{p_{r^*} + p_g' - p_L}, \]
(19)
with \( p_g' \) denoting the partial pressure of the gas within the critical size nucleus. Note that the contaminant gas always acts as to reduce the critical radius. Although no gas bubble can, in principle, exist in equilibrium in a liquid unsaturated with the gas and will eventually dissolve and disappear when the liquid, the possibility of existence of Harvey nuclei, for which the gas is trapped in a crevice in a particle either due to the highly convex curvature or due to the sufficient elasticity of the bounding surface.

We now define the cavitation onset point with local pressure \( p_{c^*} \) as the location where sufficient nucleation sites are activated for observed or detectable cavitation. If we assume that the partial gas pressure obeys Henry’s law (\( p_g' = H c' \)) at the onset of cavitation, with \( c' \) denoting the concentration of dissolved gas and \( H' \) denoting Henry’s constant, the cavitation onset bubble radius \( R_c' \) (the critical radius at the onset of cavitation) now follows from eq. (19) as
\[ R_c' = \frac{2S'}{(p_{r^*} - p_L)(1 + \beta)} \]
(20)
where
\[ p_{g^*} = p_{r^*} + p_g' - p_L. \]
The normalized nucleation rate, introduced in eq. (21), is the ratio of the partial gas pressure inside the bubble to the pressure difference between the vapor phase and the liquid at the onset of cavitation. The steady-state nucleation rate following the cavitation onset point can now be evaluated by eqs. (19)-(21) as

\[
J' = Z_i \left[ \frac{2S' \rho_i^2}{\rho \tau_i} \right]^{1/2} \exp \left\{ -\frac{16\pi^\alpha(1-2\alpha)}{3k_B T_i' \left[ p_i' - p_L + \delta(p_i' - p') \right]^2} \right\}
\]  

with \(0 \leq \alpha < 1/2\). The local critical radius following the cavitation onset point is now given by

\[
r_c' = \frac{2S'}{p_i' - p_L + \delta(p_i' - p')}
\]  

STEADY-STATE NOZZLE FLOWS WITH NUCLEATION

We now restrict the cavitating flow model with nucleation to quasi-one-dimensional steady-state nozzle flows in order to compare the results arising from nucleation with those of non-nucleating bubbly flows discussed by Wang and Brennen[1] and by Delale et al.[2]. We further neglect the compressibility of the liquid by taking the limit \(M \to 0\) and \(\rho_v \to 0\). The normalized quasi-one-dimensional steady-state cavitating nozzle flow equations then take the form

\[(1 - \beta)uA = 1 - \beta,\]  

\[
du/dx + u \left( \frac{1}{A} \frac{dA}{dx} \right) - 3 \beta R \frac{dR}{dx} - \frac{4}{3} \pi \rho^3 R^3 J = 0,
\]

where

\[
J' = \frac{\rho_i^2}{\rho \tau_i} \left[ \frac{2S' \rho_i^2}{\rho \tau_i} \right]^{1/2} \exp \left\{ -\frac{16\pi^\alpha(1-2\alpha)}{3k_B T_i' \left[ p_i' - p_L + \delta(p_i' - p') \right]^2} \right\}
\]

\[
\frac{u^2}{R} \frac{d^2R}{dx^2} + u \frac{dR}{dx} \left( \frac{dR}{dx} \right)^2 + \frac{3}{2} u^2 \left( \frac{dR}{dx} \right)^2 = \frac{1}{\pi^2} p_g - p - \frac{4u}{(Re)R} \frac{dR}{dx} - \frac{2}{(We)R}
\]

where \(\beta_i\) denotes the inlet void fraction (for pure liquid flow at the inlet \(\beta_i = 0\)). The normalized nucleation rate \(J\) in eq. (25) can be written as

\[
J = J_0 \exp \left\{ -Gb \right\}
\]

the pre-exponential factor \(J_0\) and the normalized Gibbs activation function are given by

\[
J_0 = \frac{H_i^4}{U_i} Z_i \left[ \frac{2S' \rho_i^2}{\rho \tau_i} \right]^{1/2}
\]

and

\[
Gb = \frac{16\pi^\alpha(1-2\alpha)}{3k_B T_i' \left[ p_i' - p_L + 2S' / R_i^2 \right]^2}
\]

In arriving at eq. (30), we have eliminated \(\delta\) between eqs. (20) and (22). It is important to mention that, in the nucleation zone following the onset of cavitation, a single normalized bubble radius \(R\) is used in the Rayleigh-Plesset equation (27) to describe the growth of the bubbles produced upstream as well as the newly nucleating embryos. It should also be mentioned that, until now, the bubble pressure \(p_g\) has been left quite arbitrary. It is well-known that the pressure inside the bubble affects bubble dynamics significantly (Nigmatulin et al.[11], Prosperetti[3], Matsumoto and Takemura[12]) and requires simultaneous consideration of the energy and transport equations within the bubble and in the surrounding liquid. For cavitating flows, the coupling of these equations with the model equations makes the solution formidable. Therefore, approximations are introduced. A simple model that discusses the thermal damping problem in cavitating nozzle flows , following the work of Prosperetti[3], is given by Delale[4]. In this work, however, we will use the polytropic law for the gas, solely, to be able to compare our results of steady-state cavitating flows with those of bubbly cavitating flows of Wang and Brennen[1]. Consequently, we use

\[
p_g = p_v + p_g = p_v + \frac{p_{ng}}{R^\kappa}
\]

where \(p_{ng}\) denotes the normalized partial gas pressure at the inlet for bubbly cavitating flows and the normalized partial gas pressure at the onset of cavitation for cavitating flows with nucleation and \(k\) is the polytropic index. By eliminating \(\beta\) between eqs. (24)-(26), we obtain

\[
\frac{uA - \beta_i}{R^\kappa} = \beta_i + \frac{4}{3} \rho^3 \int_{x_i}^x J(x) \mu(x) d\xi
\]

and

\[
\frac{du}{dx} + A \frac{dp}{dx} = 0
\]

where we have defined the normalized axial coordinate \(x\) to denote the cavitation onset point for which \(J\) practically vanishes if \(x < x_i\). In particular, for bubbly cavitating flows with an inlet void fraction \(\beta_i\), eq. (32) reduces to

\[
\frac{\beta}{(1 - \beta)R^{\kappa}} = \frac{\beta_i}{(1 - \beta_i)} = \text{constant}
\]

as required in the formulation of quasi-one-dimensional bubbly nozzle flows (see van Wijngaarden[13], Wang and Brennen[1], Delale et al.[2]). On the other hand, for the flow of pure liquid with cavitation onset at \(x = x_i\), eq. (32) takes the form

\[
\frac{uA}{1 + \frac{4}{3} \rho^3 R^3 \int_{x_i}^x J(x) \mu(x) d\xi} = \int_{x_i}^x J(x) \mu(x) d\xi.
\]

We now define

\[
E_i(p, R, x) = \frac{u}{(1 - \beta_i)A} + \left[ \beta_i + \frac{4}{3} \rho^3 \int_{x_i}^x J(x) \mu(x) d\xi \right] \frac{R^3}{A}
\]

the p-dependence of the functional \(E_i\) arising from the p-dependence of the normalized nucleation rate \(J\). It then follows by direct differentiation that

\[
E_2(p, R, x) = \frac{du}{dx} = \frac{(1 - \beta_i) dA}{A^2 dx} + \frac{4}{3} \rho^3 JR^3
\]
whereas the correction factor (only for \(3/\bar{\varepsilon} \leq \bar{\varepsilon}\)) is varied, defined by \(\phi\). Here, the nucleation rate first reaches a peak, a quasistatically stable cavitating, similar to those obtained by Wang and Brennen [1] \(\phi\) so that cavitation occurs inside the nozzle. For \(\phi\) for the Gibbs activation energy, the and \(\beta\)) along the \(\phi\) implying liquid flow without bubbles at inlet. The inlet and 14.2 and 0.1 \(\varepsilon\) for the number \(\phi\) can now be written as \(\phi\) and the Reynolds number \(Re\) ) and the polytropic equations contains the parameters \(37\), respectively. For a given fluid, the above system of \(\phi\) at 20 C (with 1000 \(\rho_i\) = 1000 kg/m\(^3\) , \(\mu_i\) = 1\times10\(^{-3}\) Ns/m\(^2\), \(S'\) = 0.074 N/m , \(p_i'\) = 0.0234 bar). For reasons of comparison with the steady-state bubbly cavitating nozzle flow solution of Wang and Brennen[1], we employ the geometric nozzle configuration of Wang and Brennen, as shown in fig. 1. By appropriate scaling, we can write the normalized nozzle area of Wang and Brennen as

\[
A(x) = \left[1 + 0.5(1 - \cos(2\pi x/5))\right]^{1/2} \text{ for } 0 \leq x \leq 5
\]

and unity elsewhere where the origin of the normalized axial coordinate is chosen such that the throat is located at \(x = 2.5\). The correction factor \(\alpha\) for the Gibbs activation energy, the critical radius \(R'\) at the onset of cavitation and \(\delta\), defined by eq. (21), are chosen as parameters in the model since there are uncertainties in determining these quantities. As for the nozzle inlet conditions, the inlet void fraction is set equal to zero (\(\beta = 0\)) implying liquid flow without bubbles at inlet. The inlet flow speed and the inlet cavitation number \(\sigma\), defined as

\[
\sigma_i = \frac{p_i' - p_i}{0.5 \rho_i' U_i'^2}
\]

with \(p_i'\) denoting the inlet pressure, are set equal to \(U_i' = 10\) m/s and \(\sigma = 0.8\) so that cavitation occurs inside the nozzle. For direct comparison with the results for bubbly flows of Wang and Brennen, an effective viscosity, 30 times as much as that of water, is used to account for various damping mechanisms in a crude manner. The cavitation onset point is determined by varying the critical bubble radius \(R'\) between 9 and 20 \(\mu m\) for \(\delta = 0.5\) and \(\delta = 1.0\) whereas the correction factor \(\alpha\) is varied critically close to the limiting value 0.5 within ten decimal places. The micro to macro scale is set equal to \(\ell = 10^{-3}\).

The integro-differential system of equations (38)-(40) were solved under the above specific conditions using a fourth-order Runge-Kutta method with adaptive step size. The results are shown in figures 2 and 3. Figure 2 (a)-(e) show quasi-statically stable as well as unstable flow solutions for \(\sigma = 0.4999999927\) and \(\delta = 0.5\), similar to those obtained by Wang and Brennen[1] for bubbly flows. When the initial critical radius changes between 14 \(\mu m\) and 14.2 \(\mu m\), a quasistatically stable cavitating flow regime with a ringing structure downstream of the nozzle \((x > 5)\) is observed. Figures 3 (a)-(d) show, respectively, the distributions of the normalized flow speed \(u\), the pressure coefficient \(C_p\), defined as

\[
C_p = \frac{p' - p_i'}{0.5 \rho_i' U_i'^2}
\]

the normalized radius \(R\) and the void fraction \(\beta\) along the nozzle axis. The flow speed fluctuates with an amplitude of about 10 % of the inlet flow speed in the constant area zone downstream of the diverging section of the nozzle \((x > 5)\) and eventually returns to its upstream value due to the various bubble damping mechanisms (here taken into account, in an adhoc manner, by an effective viscosity). The nucleation rate, as shown in fig. 2 (e), peaks at the throat with a value of \(J'_{\text{max}} = 10^{10} \text{ m}^{-3}\text{s}^{-1}\). For the same values of the parameters and under the same flow conditions, a bifurcation to a ‘quasi-statically unstable flow’ (or ‘a flashing flow’) occurs around \(R' = 14.2 \mu m\). Here, the nucleation rate first reaches a peak and, then, diminishes, with almost no difference compared with the ‘quasi-statically stable’ case. However, the normalized radius \(R\) and the void fraction \(\beta\) continue to grow.
Figure 2 : The distributions of (a) the normalized flow speed $u$, (b) the pressure coefficient $C_p$, (c) the normalized bubble radius $R$, (d) the void fraction $\beta$ and (e) the nucleation rate $J'$ [$m^3/s$] along the axis of the nozzle used by Wang and Brennen[1] under the flow conditions with the inlet flow speed $U'_i = 10$ m/s, inlet cavitation number $\sigma_i = 0.8$ and nucleation rate parameters $\alpha = 0.49999999927$ and $\delta = 0.5$ for the values of the critical bubble radius at the onset of cavitation $R'_i = 14.0$, 14.2 and 20.0 $\mu$m (a stable cavitating flow solution with a downstream ringing structure is shown for $R'_i = 14.0$ $\mu$m).
Figure 3: The distributions of (a) the normalized flow speed $u$, (b) the pressure coefficient $C_p$, (c) the normalized bubble radius $R$, (d) the void fraction $\beta$ and (e) the nucleation rate $J'$ [m$^{-3}$s$^{-1}$] along the axis of the nozzle used by Wang and Brennen[1] under the flow conditions with the inlet flow speed $U'_i = 10$ m/s, inlet cavitation number $\sigma_i = 0.8$ and nucleation rate parameters $\alpha = 0.4999999930$ and $\delta = 1.0$ for the values of the critical bubble radius at the onset of cavitation $R'_c = 9.0$, 9.1 and 14.0 $\mu$m (a stable cavitating flow solution with a stationary bubbly shock wave is shown for $R'_c = 9.0$ $\mu$m).
Consequently, the pressure can not recover due to the exponential growth of the void fraction outweighing the effect caused by local area change in the diverging section of the nozzle. The pressure suddenly drops resulting in a second nucleation zone where the nucleation rate now grows without limit. As \( R' \) is further increased (see fig. 2 for \( R'_i = 14.2 \mu m \)), the nucleation rate grows exponentially and without limit after the onset of cavitation, causing instabilities in the flow field near the throat.

A cavitating flow with a bubbly shock wave also seems possible in this model under the same flow conditions for the values of the parameters \( \delta = 1.0 \) and \( \alpha = 0.4999999930 \), when the initial bubble radius \( R'_i \) is increased beyond \( 8 \mu m \). Figure 3 shows the results of cavitating nozzle flows with a stationary bubbly shock wave in the divergent section of the nozzle for \( R'_i = 9.0 \mu m \). The nucleation rate now peaks at \( J'_{max} = 10^{11} m^{-3}s^{-1} \) (fig. 3(e)). The distributions of the normalized flow speed \( u \), the pressure coefficient \( C_p \), the normalized radius \( R \) and the void fraction \( \beta \) along the nozzle axis are shown in fig. 4 (a)-(d). The normalized radius relaxes towards unity after a few rebounds and the pressure recovers very sharply towards the inlet pressure, but with some pressure loss due to dissipation. Under the same conditions and fixed values of the parameters \( \delta \) and \( \alpha \), if the initial radius is further increased, a bifurcation to a quasi-statically unstable solution with a sudden pressure drop in the divergent section of the nozzle (consequently, a second nucleation front with the nucleation rate growing without limit) occurs, similar to the ‘flashing flow’ solutions observed by Wang and Brennen[1]. Increasing the initial radius at the onset of nucleation leads to the movement of the instability towards the throat, as shown in fig. 3 for \( R'_i = 14.0 \mu m \). In this case the instability occurs near the throat where the pressure drops and the flow speed increases almost discontinuously with the nucleation rate growing without limit. The computation for the radius breaks down in this case due to very high growth rates. The results of this investigation show similar steady-state patterns of homogeneous bubbly flows, but with higher growth and collapse rates due to bubble formation by nucleation.

CONCLUSIONS

Quasi-one-dimensional steady-state cavitating nozzle flows are considered by taking into account the effect of bubble nucleation. For this reason, the onset of cavitation is modeled by using an improved version of the classical nucleation rate equation and by treating the unknown quantities like the initial partial gas pressure and the initial bubble radius at the onset of cavitation as parameters. Therefore, a parametric investigation of the onset of cavitation in quasi-one-dimensional cavitating nozzle flows is carried out. The nonlinear dynamics of cavitating bubbles is described by the classical Rayleigh-Plesset equation where a polytropic law for the partial gas pressure is employed by taking into account the effect of damping mechanisms by an effective viscosity in a rather crude manner, for a direct comparison with the results obtained by Wang and Brennen[1] for homogeneous bubbly flow. The results of this investigation show similar characteristics of homogeneous bubbly flows, but with intensified growth and collapse. Stable steady-state cavitating nozzle flow solutions as well as quasi-statically unstable flow solutions are found. In particular, a relatively thin nucleation zone followed by bubble growth and collapse zones can be distinguished for stable steady-state solutions. Moreover, when the parameters entering the nucleation rate equation are properly chosen, stable steady-state cavitating nozzle flow solutions with stationary bubbly shock waves in the diverging section of the nozzle seem possible. We should finally mention that the unstable solutions, encountered here, can be stabilized by taking into account thermal damping (see Delale[4]) and/or flow unsteadiness (see Preston et al. [5]).

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