The formation of cavity clusters at sheet cavity / re-entrant jet contact

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ABSTRACT

The fast flow of water past a hydrofoil often results in the formation of a sheet cavity on the suction side of the foil, and from its trailing edge a re-entrant jet develops and moves upstream into the cavity. Eventually this jet gets into contact with the cavity surface and causes the formation of small clouds of cavities at the positions of contact [1-2]. To explain these observations, the principal parameters expected to be responsible for the cloud cavitation are used in a simplified theoretical model. Experiments based on this model confirm the theoretical approach. The results show that earlier speculations that the surface tension $\gamma$ should be a radius-dependent quantity are not correct [3].

INTRODUCTION

A sheet cavity develops by flow detachment close to the leading edge of a hydrofoil and grows into an extended, thin vapour cavity with a very smooth water-vapour interface along the suction side of the foil, Figure 1. It terminates at some downstream position of the foil, along a line of flow re-attachment. The associated deflection of the flow implicates the occurrence of a locus of stagnation on the foil. The thin layer of water flowing between the cavity surface and the sheet of stagnating streamlines is deflected into the sheet cavity as a re-entrant jet. This jet moves upstream along the foil surface towards the leading edge of the sheet cavity, gradually filling it. During this process it comes into contact with the surface of the sheet cavity at singular points or along line segments. At locations where this occurs, clouds of cavities are formed. This is illustrated in Figure 2 which shows four subsequent high speed photographs of a 120 mm hydrofoil with a sheet cavity extending from the leading edge (the flow is in the downwards direction) and reaching its maximum at about 70% of the foil width. The frames are taken with an inter-frame time difference of about 2 ms. The front of the re-entrant jet is seen as a slightly wavy bright line above the curved trailing edge of the sheet cavity. In the first frame a big (double) cavity cloud is already located at the maximum extension of the sheet cavity, and a tiny

Figure 1. A hydrofoil with a sheet cavity into which a re-entrant jet penetrates.
cloud has just developed at point A, behind the front of the re-entrant jet. In the second frame this cloud has grown, and a new one has emerged at B. In the third frame both of these clouds have expanded while also moving transversely towards the big cloud. We observe that to the right of the big cloud, at about 80% of the foil width, a new cloud has developed just behind the leading edge of the re-entrant jet. In the fourth frame the cloud at B is now merging with the big cloud. The two remaining, small clouds have developed further, and are approaching the big cloud, which eventually swallows them. The small clouds arise at contact between the re-entrant jet and the sheet cavity. It is the object of the present paper to analyse the mechanism of formation of this type of cavity cloud.

THEORY

At contact between a sheet cavity and a re-entrant jet, two oppositely directed flows of high velocity merge. The zone of merging expands radially from the initial point of contact due to surface tension forces at its boundary. Inside the contact zone the flow velocities set up a high shear stress, but at the boundary, this shear stress is taken to be a factor of secondary influence only.

Neither the flow velocities nor the shear stress are included in the model considered in this paper. It focusses on the effects of the surface tension, and the process of merging is modelled theoretically by considering the contact of two regions I and II of stationary water, initially separated by a layer of water vapour and non-condensable gas at low pressure, Figure 3a. The liquid in the lower region I is taken to have a planar liquid-vapour interface. The upper region II has a non-planar liquid-vapour interface of large, positive radius of curvature $S$ (that may depend on location), its lowest $z$-position being at $(x,y)=(0,0)$. Region II is made to move towards region I ($z$-direction) with a negligible velocity. At time $t=0$, the regions I and II get into contact at $(x,y)=(0,0)$. Before contact occurs, the gas and vapour pressures are $p_e=p_g+p_v \approx 0$ in the whole system. When contact is established, the merging zone of the regions I and II becomes bounded by a closed-loop contact surface around $(x,y)=(0,0)$, which in vertical cross sections has a very high negative maximum curvature $\kappa_v$ (concave). In horizontal planes, the loop curvature $\kappa_h$ is positive (convex) and at the moment of contact also very high, but due to the flatness of the two merging surfaces the radius of the contact zone $R_h$ expands rapidly, and $\kappa_h$ becomes much smaller than $|\kappa_v|$. In the liquid just behind the
surface of merging, the surface tension $\gamma$ changes the local pressure into

$$P_I = p_\infty + \gamma(\kappa_b + \kappa_i) = p_\infty + \gamma(1/R_b + 1/R_i). \tag{1}$$

Figure 3. a) Two water regions, $I$ and $II$, separated by a layer of non-condensable gas and vapour. b) The merging of the two regions at contact. c) The volume $dV_{merge}$ of the sink, supplied from the sources by the volumes $dV_I$ and $dV_{II}$, thus setting up the convection of the boundary of the merging zone in the time $dt$.

Therefore, the closed-loop contact surface constitutes an annular line sink which is supplied with liquid from the neighbouring, compliant liquid/gas-vapour interfaces of $I$ and $II$, where the pressure is $p_\infty$. Thus, closed-loop line sources, each of half the strength of the sink, are located here. Figure 3b shows a cross section of the region of merging shortly after the moment of contact. The sink-source system actually constitutes a closed-loop line dipole that bounds the merging zone, expanding at the speed $U_i, i=A,B$ (and equivalent points in all other vertical cross sections) in dependence of the local distance $h_i$ of the merging surfaces, measured at the transition from sink to sources, i.e. the maximum distance between the locally deformed surfaces of the regions $I$ and $II$. This distance governs the dipole strength. In the very first instant of contact, when it is on a single molecular scale only, and $\kappa_b \approx |\kappa_i|$, resulting in $P_I = p_\infty$, the expansion of the merging zone is governed by the approach velocity of $II$ to $I$, but as soon as the contact zone extends over just a few molecules, $\kappa_i$ dominates the strength of the sink, and a dipole flow field is established. Now, the approach of $II$ to $I$ becomes of no importance. If the tensile strength of the liquid is exceeded, cavities are created, and they grow to relax the stress field set up by the sink.

Let us limit our considerations to cylindrical symmetry around $(x,y)=(0,0)$, and the closed-loop line dipole becomes circular. During the very first moments of merging, the dipole is highly varying in time, but subsequently it can be considered quasi-steady. The surface energy being released by the expansion of the merging zone, i.e. by reduction of the surface area, is converted into the pressure difference $p_\infty - P_I$ driving water from the two sources into the sink. Figure 3c shows the displacement of the merging zone within a small interval of time $dt$, and reveals the location of the sources and the sink, and the volumes of water that are moved from the former to the latter. Thus, we can write

$$2\gamma U_i dt = (p_\infty - P_I) h_i U_i dt,$$

which with $p_\infty - P_I = \gamma \rho U_i^2$ gives

$$U_i^2 = 4\gamma/(\rho h_i). \tag{2}$$

The sink can be interpreted to be a fraction $q$ of a simple, ideal line sink of core radius $R_i$, being convected outwards in a horizontal, radial direction (together with the associated sources). The mass of liquid, which enters this sink, and its kinetic energy, must equal the mass and the kinetic energy of the liquid that enter the real sink moving into the gas-vapour layer. The thickness of the vapour layer $h_i$ at the contact surface determines the core radius $R_i$ of the sink, which has the strength per unit length $Q_i = h_i U_i = q\pi R_i$, $V_i$, $V_r$, is the radial velocity at the core of the ideal sink. The energy equality gives $h_i U_i^3 = q\pi R_i V_r^3$, and therefore $U_i = V_r$. Thus,

$$h_i = q\pi R_i. \tag{3}$$

The pressure $P_I$ in the liquid at the core of the ideal sink, as well as its minimum value at the real sink, therefore becomes

$$p_\infty - P_I = \gamma/R_i = q\pi\gamma/h_i, \tag{4}$$

which leads to

$$q = 2/\pi. \tag{5}$$

Real liquids contain lots of particles which supply cavitation nuclei, and at sufficiently low pressure a number of them grow into supercritical cavities, a process that affects the velocity and pressure distributions in the liquid. If the region $II$ has an appreciable surface curvature, the rapid increase of $h_i$ limits the occurrence of a significant pressure drop to a small central zone around the initial point of contact, and only a few cavities can be expected to develop. However, they may grow to a relatively large size as the merging proceeds. If the curvature is small, $h_i$ increases only slowly with distance from the initial point of contact, and the formation of an extended, thin cavity cloud – a sheet cloud - can be expected. When the cavities in this cloud have become supercritical they tend to relax the stress field behind the convected
dipole, and new cavities developed at the sink collapse as the sink proceeds outward.

**EXPERIMENTS**

A closed PMMA container of quadratic cross section was filled with water to a level slightly below a planar, horizontal solid surface. This could be either an 8.0 mm diameter brass rod, or the end surface of a wooden pole inlaid in a 25.0 mm diameter brass holder mounted on the 8 mm rod, Figure 4. By means of an adjustment nut the solid surface was then lowered until contact with the planar water surface was obtained. Subsequently it was retracted until contact was broken. This caused a water drop to cling to the solid surface, Figures 5a, b.

![Figure 4. The experimental set up.](image)

In Figure 5a we see that the brass surface was carrying a drop with a diameter of 3.9 mm and a height of 1.1 mm. Its static wetted angle of attachment was $\theta \sim 50^\circ$, i.e. the surface was moderately hydrophilic. The initial contact of the dry brass surface with the water surface always caused the formation of a few gas bubbles, which stabilised inside the drop after its formation. Therefore, at contact between the drop and the planar water surface below, the large bubbles already present prevented stabilisation of new cavities, nucleated by the merging. However, the merging was itself an object of investigation.

The wooden surface of the 25 mm holder was soaked with water before being used. As apparent from Figure 5b, it was highly hydrophilic, and when it was retracted from the water surface, a wide drop which smoothly joined with the wooden surface, $\theta \rightarrow 0^\circ$, was formed, with a height of only 0.62 mm. This drop did not stabilise gas bubbles in its interior after separation from the water surface.

After having produced the drop, it was lowered slowly until contact with the planar water surface, thus initiating their merging. The moment of contact was identified by the closing of an electrical circuit connected to the volume of water in the container and to the adjustable brass rod. This circuit contained a delay line, which generated a pulse that fired a flash lamp at a pre-set time after the moment of contact. In this way, single pictures of the process of merging were obtained (flash duration 3 µs at 1/3 of the peak intensity), Figures 6 and 9.

![Figure 5. A drop of water hanging a) below the 8 mm brass rod just above the planar water surface (oblique view). $\theta = 47^\circ/54^\circ$, b) below the 25 mm wooden surface (horizontal view). $\theta \sim 0^\circ$.](image)

*With the 8 mm brass rod carrying a relatively high drop (Figure 5a), the merging was observed at an angle of $\sim 3^\circ$ above the horizontal plane, thus revealing the shape of the contact dipole and its convection in the radial direction. This is illustrated in Figure 6, where (a) shows the drop and its mirror image 0.5 ms after the moment of contact. A merging zone of a diameter of 1.3 mm has developed. The distortion of the initial drop shape and of the planar water surface, related to the sources of the dipole, are visible just ahead of the contact surface. In (b), taken 3.0 ms after contact, the merging zone diameter has grown to 3.0 mm. Now, the original drop is almost swallowed into the merging zone, and the source located at the surface of the drop is just about to be annihilated. This causes a reduced (non-equilibrium) wetted contact angle, and the lower source takes over full responsibility for supplying water to the sink. The theory above assumes the presence of both sources for determining the sink height $h_i$. When the upper source dries out, $h_i$ may be calculated as twice the difference of the z-coordinates of the point on the surface of merging, where its tangent is vertical, and that of transition from source to sink on the lower, free water surface, Figure 7. The surface of merging has a vertical tangent only, if part of the drop is still left, or if the surface
carrying the drop is fully wetted so that the liquid can advance freely, i.e., if it is totally hydrophilic. In (c) the drop is totally absorbed in the merging zone (the upper liquid region \( \text{II} \) has disappeared) which now advances by liquid-solid contact. The locus of attachment is dragged to larger radii at non-equilibrium wetted angles of contact larger than \( 90^\circ \). This cannot be calculated from the theory above. The radius of the merging zone vs. time \( \tau \) after contact is shown in Figure 8a until a point where annihilation of the drop is approaching. In Figure 8b, experimental and calculated growth velocities, \( (U)_\text{exp} \) and \( (U)_\text{calc} \) are shown.

\[
h_i \approx 2.5 \Delta h .
\]  

Figure 6. The process of merging below the 8 mm brass rod a) 0.5 ms, b) 3.0 ms, and c) 12.5 ms after contact at \( p_c = 0.18 \) bar, observed from an angle of \( \sim 3^\circ \) above the horizontal plane.

Figure 7. Determination of the vapour layer thickness, \( h_i \), when the upper source dries up.

A comparison of Figure 5a with the photographs of the merging process, Figure 6, shows that during the merging, \( h_i \) is related to the local vertical distance, \( \Delta h \), between the undisturbed surfaces of the drop and the water below by

\[
h_i \approx 2.5 \Delta h .
\]  

Figure 8. a) The experimental radius \( R_h \) of the merging zone vs. the time delay \( \tau \) for the drop in Figure 6 (until its annihilation approaches). b) The growth velocity \( (U)_\text{exp} \) determined from the fitted curve in Figure 8a, and the one calculated from (2), \( (U)_\text{calc} \), using experimental values of \( h_i \) (see Figure 6).

The merging of a drop attached to the 25 mm holder with the wooden surface (Figure 5b) was studied at an angle of \( \sim 20^\circ \) below the horizontal plane. The flash lamp was placed at the opposite side, illuminating the merging zone from angles of \( \sim 30^\circ \) below the horizontal plane. Thus, the merging zone appeared as an ellipsoidal structure on the totally reflecting planar water surface, and with the solid surface as background in its interior. Each bubble showed up with a bright spot due to light transmitted from the flash lamp. It was found that this merging led to the formation of a planar cavity cluster in the central merging zone as illustrated in Figures 9a,b,c.

These pictures were taken 1 ms, 23 ms and 35 ms after the moment of contact, respectively, at \( p_c = 0.14 \) bar. (Cavities could be
generated at pressures up to $p_\infty \approx 0.3$ bar, which reflects the low tensile strength of the liquid. In Figure 9b we notice that the reduced pressure along the closed-loop line sink has caused the formation of numerous micro-cavities (indicated by the arrow), large enough to be visible as bright bubble spots. These were too small to survive the pressure increase when the sink continued its motion in outward, radial direction. In Figure 9c the merging zone has reached the outer diameter of the 25 mm holder with the wooden surface, where the sink strength decreases towards zero, and no micro-cavities are observed – only those in the central area of merging remain. At the pressure used, even these cavities shrink and dissolve by time, which makes it possible to repeat the

Figure 9. Merging zone with a cavity cluster below the 25 mm wooden surface a) 1 ms, b) 23 ms, and c) 35 ms after contact at $p_\infty = 0.14$ bar.
experiment without having stabilised bubbles in the drop (Figure 5b).

In Figure 10a the radius $R_h$ of the merging zone is shown vs. the delay time $\tau$ as obtained from a series of photographs. Its velocity $(U_i)_{\text{exp}}$, determined from the fitted experimental curve, is shown in Figure 10b together with the velocity calculated from (2), $(U_i)_{\text{calc}}$, by applying (6) to $\Delta h$-values obtained from Figure 5b.

![Figure 10](image)

**Figure 10.** a) The radius of the merging zone vs. time $\tau$ after the contact of a drop, hanging below the 25 mm wooden surface, with the free water surface underneath. b) Velocity of convection $U_i$ calculated from the fitted line in (a) (the locations of measurement are indicated), and from Equation (2).

**SUMMARY**

The theory presented above is based on the surface tension of a liquid $\gamma$ being constant. Thus, we reject the proposal that $\gamma$ should be radius dependent at small radii of curvature, as argued by Atchley [3] in an attempt to support the claim that free micro-gas bubbles in water were stable. If, as suggested in [3], $\gamma \to 0$ for the radius of curvature of a liquid surface $R \to 0$, then the merging of two stationary, horizontal surfaces of water being brought into contact would be prevented by the horizontal component of (2), $(\gamma / R)$, dominating the vertical one, $|\gamma / R_h|$. This would make the contact zone shrink – a conclusion that is in conflict with the findings of the present paper as well as with general experience. Free gas bubbles in water are inherently unstable and dissolve [4].

The nuclei responsible for cavity formation are gas nuclei stabilised at solid surfaces [5-6-7-8]. Atchley’s postulate would also imply that the breaking of a single bond in the bulk of an ideal liquid would cost an infinitesimal amount of work only, and that the breaking of more bonds would gradually increase the amount of work per bond. This is not realistic.

The experimental results obtained in the present paper show that the merging of two stationary liquid surfaces at low pressure causes the formation of a sheet cloud of cavities. This also basically explains the formation of small cavity clouds on hydrofoils at contact between a sheet cavity and a re-entrant jet as shown in Figure 2. The opposite and high, tangential flow velocities at the surface of the sheet cavity, and in the re-entrant jet, which are not treated in the present paper, presumably influence the growth and development of the cavity clusters. The strong shear stress in the merging zone sets up vorticity, which results in complex local velocities and low-pressure cells. These may stabilise the cavities that are nucleated, and may cause transverse spreading of them, so that thicker clouds are formed, as observed experimentally on hydrofoils.

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