ULTRASOUND FOCUSING IN SPHERICAL BUBBLE CLOUD

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ABSTRACT

HIFU (High Intensity Focused Ultrasound) applications have attracted much attention recently because they are less invasive. However, the behavior of acoustic cavitation caused by high pressure amplitude at the focal region has not been clarified. In this study, it is assumed that the acoustic cavitation forms a spherical bubble cloud which consists of many micro bubbles, and ultrasound focusing in the cloud is investigated numerically. The compressibility of liquid, the evaporation and condensation of liquid at the bubble wall, heat transfer through the bubble wall are considered in the simulation. Initial cloud radius is equal to 0.5 mm, bubble radius is 1 µm, void fraction is 0.1 %, and the frequency of ultrasound extends from 5 kHz to 10 MHz for amplitudes of 10 kPa, 25 kPa, 50 kPa, 75 kPa, 100 kPa or 125 kPa. The pressure wave focuses to the center of the cloud and the pressures inside bubbles increase extremely when the frequency is near the 1st mode frequency of the cloud estimated from a small amplitude approximation. Especially, in the case of 100 kPa and 125 kPa pressure amplitudes, the high pressures appear even when the frequency is much lower than the 1st mode frequency.

INTRODUCTION

Acoustic cavitation has important roles in HIFU applications. Its violent collapse has a potential of inducing tissue traumas [1]. On the other hand, it has been shown that the high pressure contributes the stone comminution in SWL (Shock Wave Lithotripsy) [2] and a method which efficiently controls the cavitation erosion only at the surface of a renal stone by two frequencies ultrasound has been developed [3]. In HIFU therapy, in which the lesion is formed by the energy of ultrasound, the contribution of the acoustic cavitation to the tissue heating has been investigated and recently utilized [4]. However, it is necessary to comprehend the dynamics of the acoustic cavitation to utilize the high pressure and temperature more efficiently.

Many researchers have investigated the dynamics of a bubble cloud numerically and experimentally [5-8]. Shimada et al. investigated the behavior of cavitation cloud in connection with the severe cavitation damage using the set of governing equations for the spherical bubble cloud, where the internal phenomena of each bubble and the compressibility of liquid are taken into account [7]. An inwardly propagating shock wave is formed during the collapse of the bubble cloud and focused in the cloud center.

In this paper, it is assumed that the acoustic cavitation forms a spherical bubble cloud which consists of many micro bubbles by HIFU, and ultrasound focusing in the cloud is numerically investigated by applying Shimada’s model [7].

THE MODEL OF A BUBBLE CLOUD

Inside a bubble cloud, shock wave propagates in bubbly flow. Kameda et al. clarified that internal phenomena have much influence on shock wave propagation [9] and in certain cases the compressibility of liquid cannot be ignored depending on state of bubble motion [10]. It is said that a very high pressure of O(10⁸)~O(10⁹) Pa emerges near the center of cloud cavitation when it collapses violently. In such a case compressibility of liquid must be taken into account. Therefore, to analyze collapsing phenomena of a bubble cloud precisely, internal phenomena of a bubble and liquid compressibility are needed. Considering such phenomena, Shimada et al. numerically simulated a bubble cloud. They concluded that when bubble cloud collapses, very high pressure emitted from each of the bubbles near the center of the bubble cloud and the variation of pressure is very high frequency [7].
Bubbles in the cloud shrink violently so that gases in a bubble cannot be treated as ideal. For simplification, ambient temperature rise and mass transformation of non-condensable gases were not considered. To sum up, in the numerical simulation, the following assumptions are employed: (1) The bubble cloud and each bubble oscillate maintaining spherical symmetry. (2) Bubbly liquid inside the cloud is treated as a continuum fluid, whose mass and momentum are assumed to be equal to those of the liquid phase, because the mass in the unit volume of gas phase is much smaller than that of liquid phase. (3) Bubbles move with the surrounding liquid. Bubbles are small enough to ignore the slippage between the bubble and the liquid. (4) Coalescence and fragmentation of bubbles in the cloud are ignored. (5) Viscosity of bubbly mixture is ignored in the cloud because it has little influence on the wave phenomena. (6) The temperature of the liquid in the cloud is constant. (7) The pressure and temperature inside the bubble are uniform except for the thin boundary layer near the bubble wall, which is thin compared with the bubble radius. (8) Temperature at the bubble wall is equal to that of liquid. (9) Mass of non-condensable gas inside a bubble is constant. (10) Gases inside a bubble obey the van der Waals gas law. (11) Coalescence and fragmentation of mist inside a bubble are ignored. Then we consider the thermal behavior inside the bubble and the pressure wave phenomena in the bubble cloud, namely, the evaporation and condensation of liquid at the bubble wall, heat transfer through the bubble wall and the compressibility of liquid.

The equation of motion of the spherical bubble cloud interface is the Keller equation [11] in which the compressibility of the surrounding liquid is taken into account as shown below.

\[
R \left(1 - \frac{\dot{R}}{c} \right) \dot{R} + \frac{3}{2} \left(1 - \frac{\dot{R}}{3c} \right) \dot{R}^2 = \frac{1}{\rho_l} \left(1 + \frac{\dot{R}}{c} \right) \frac{d}{dt} \left(p_c - p_v - 4 \mu \dot{R} \frac{R_c}{R} \right) \tag{1}
\]

\(R_c\): the initial radius of the bubble cloud, \(c\): the speed of sound in surrounding liquid, \(\rho_l\): the density of the liquid, \(p_v\): the pressure at the surface of the cloud, \(p_c\): the ambient pressure, \(\mu\): the viscosity of the liquid. Volumetric change of a bubble is solvable if pressure at the surface of the cloud is given. In a bubble cloud, mass conservation equation, momentum conservation equation, of bubbly flow and conservation equation of number density of bubbles are also solved as shown below.

\[
\frac{\partial (1-\alpha) \rho_l}{\partial t} + \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 (1-\alpha) \rho_l u_r \right) = 0 \tag{2}
\]

\[
\frac{\partial (1-\alpha) \rho_l u_r}{\partial t} + \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 (1-\alpha) \rho_l u_r^2 \right) + \frac{\partial p}{\partial r} = 0 \tag{3}
\]

\[
\frac{\partial n}{\partial t} + \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 n u_r \right) = 0 \tag{4}
\]

\(\alpha\): the void fraction, \(u_r\): the velocity of the liquid, \(n\): the number density of the bubble. The velocity of the bubble, \(u_b\), is assumed to be equal to \(u_l\) (Assumptions (3)). Tait equation is employed as equation of the liquid state.

**COMPUTATIONAL CONDITIONS**

Table 1 shows the computational conditions. An air bubble cloud in water is investigated. In medical applications, typical frequency of ultrasound is around 0.5 ~ 5 MHz. When ultrasound frequency is 4 MHz, wavelength is about 0.4 mm in water or human body. Focal region is considered to be around 2 ~ 4 times of wavelength. In this simulation we assume that region of acoustic cavitation is 0.5 mm in radius, and radius of a bubble is 1 \(\mu\)m (natural frequency is about 4 MHz).

It is assumed that the ultrasound pressure at the focal region is proportional to the acceleration of the transducers and a linear approximation is applied. Then the ambient ultrasound pressure of the bubble cloud, \(p_v\), is modeled as below.

\[
p_v = \Delta p \cdot \ddot{x} + p_0 \tag{5}
\]

\[
\ddot{x} + 2\mu \dot{x} + \omega^2 x = \begin{cases} 2\eta \sin(\omega t) & (0 \leq t \leq 4.5T) \\ 0 & (4.5T \leq t \leq 10T) \end{cases} \tag{6}
\]

\(\Delta p\): the amplitude of ambient pressure, \(p_0\): initial ambient pressure, \(\eta\): normalized damping coefficient, \(\omega\): the angular frequency of ultrasound, \(t\): time, \(T\): the cycle of ultrasound. In this simulation, \(\eta\) is equal to 0.3. Fig. 1 shows the ambient ultrasound pressure and Fig. 2 shows its spectrum.

<table>
<thead>
<tr>
<th>Computational conditions</th>
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<tbody>
<tr>
<td>Initial cloud radius, (R_c)</td>
<td>0.5 mm</td>
</tr>
<tr>
<td>Initial bubble radius, (R_0)</td>
<td>1 (\mu)m</td>
</tr>
<tr>
<td>Initial ambient pressure, (p_0)</td>
<td>101.3 kPa</td>
</tr>
<tr>
<td>Initial temperature</td>
<td>293 K</td>
</tr>
<tr>
<td>Initial void fraction</td>
<td>0.1 %</td>
</tr>
<tr>
<td>Amplitude of ambient pressure</td>
<td>10 kPa ~ 125 kPa</td>
</tr>
<tr>
<td>Frequency of ambient pressure</td>
<td>5 kHz ~ 10 MHz</td>
</tr>
</tbody>
</table>

![Fig. 1 Time variation of the ambient pressure](image1)

![Fig. 2 The FFT analysis of the ambient pressure](image2)
THE FREQUENCY RESPONSE OF A BUBBLE CLOUD

The bubbles which are located near the center of bubble cloud violently collapses when the bubble cloud oscillates at its resonance frequency. The maximum pressures inside the bubbles are calculated for various conditions. Fig. 3 shows the results. The horizontal axis corresponds to the frequency of ultrasound and the vertical axis corresponds to the maximum pressure normalized by the amplitude.

In case that the frequency is low enough, around 10 kHz, the normalized maximum pressure is nearly equal to 1. This means that the ultrasound doesn’t focus in the cloud and doesn’t interfere with each other at all at these frequencies. On the other hand, the normalized pressure converges to 0 when the frequency is around or higher than the natural frequency of the single bubble, which is about 4 MHz. This is because the bubble oscillates in opposite phase of the ultrasound and the ultrasound can’t propagate in the cloud.

In case that the frequency range is about from 170 kHz to 210 kHz, which is almost equal to the 1st mode frequency of the cloud estimated by the small amplitude approximation [6], 220 kHz, the bubble cloud resonates and the normalized maximum pressure becomes very high. Furthermore, the normalized pressure becomes higher as the pressure amplitude of the ultrasound becomes bigger due to the nonlinearity of the bubble cloud. The normalized pressure reaches 3200, which corresponds to 400 MPa, in case of 125 kPa pressure amplitude at a frequency, 170 kHz. Additionally, in the case of 100 kPa and 125 kPa pressure amplitudes, the high pressures appear even when the frequency is much lower.

In case that the frequency is high enough, around 4 MHz, the normalized maximum pressure is nearly equal to 1. This means that the ultrasound doesn’t focus in the cloud and doesn’t interfere with each other at all at these frequencies. On the other hand, the normalized pressure converges to 0 when the frequency is around or higher than the natural frequency of the single bubble, which is about 4 MHz. This is because the bubble oscillates in opposite phase of the ultrasound and the ultrasound can’t propagate in the cloud.

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Fig. 4 shows the water pressure in case that the amplitude of ambient pressure is 125 kPa at a frequency, 170 kHz. Please notice that the range of the color bar is not from minimum pressure to maximum pressure in order to attain the clear graph. The contour surface plot indicates that the water pressures change at almost same phase in the cloud. This is the 1st mode oscillation of the cloud. Fig. 5 shows the water pressure in the cloud in the same condition as the surface plot. The range of the normalized time is from 4.0 to 4.35. The ultrasound wave focuses at the center of the cloud and generates very high pressure, and the precursors appear before the focusing. Fig. 6 shows the time variation of the bubble radius and the pressure inside the bubble at the center of the bubble cloud in the same condition. The bubble gradually expands and subsequently collapses due to the focused ultrasound wave. The pressure inside the bubble reaches 400 MPa, as shown in Fig. 3, and the oscillation has a very high frequency.

![Fig. 3 Frequency response curves of the maximum pressure inside the bubbles at various amplitudes of ambient pressure](image1)

![Fig. 4 The water pressure in the case of the 1st mode of the spherical bubble cloud: The amplitude of ambient pressure is 125 kPa at a frequency, 170 kHz (contour surface)](image2)

![Fig. 5 The water pressure in the case of the 1st mode of the spherical bubble cloud: The amplitude of ambient pressure is 125 kPa at a frequency, 170 kHz (surface plot)](image3)
Fig. 6 Time variation of the bubble radius and the pressure inside the bubble at the center of the bubble cloud in the case of the 1st mode of the cloud: The amplitude of ambient pressure is 125 kPa at a frequency, 170 kHz

Fig. 7 The water pressure in the case of 2nd mode of the spherical bubble cloud: The amplitude of ambient pressure is 125 kPa at a frequency, 600 kHz

Fig. 8 Frequency response curves of the maximum bubble cloud radius at various amplitudes of ambient pressure

In Fig. 3, the peak of the pressure is also seen at 600 kHz. Fig. 7 shows the water pressure in case that the amplitude of ambient pressure is 125 kPa at a frequency, 600 kHz, which is about three times as the 1st mode of the cloud. The graph indicates that the phase of the water pressure at the center of the cloud is almost π backward from that at the boundary of the cloud. The cloud oscillates in its 2nd mode in the condition. The normalized pressure in Fig. 3 is about 60, and it is much less than the maximum pressure in case of the 1st mode oscillation.

Fig. 8 shows the frequency response curves of the maximum bubble cloud radius. The vertical axis denotes the difference between maximum radius and initial radius normalized by the initial radius. The cloud radius hardly changes in each case. This is because the changes of the cloud radius are mainly caused by the volumetric changes of the bubbles in the cloud and the initial void fraction is only 0.1 % as shown in Table 1. This is especially apparent in case that the frequency is higher than the 1st mode of the cloud.

CONCLUSIONS

The compressibility of liquid, the internal phenomena of the bubbles are taken into account, and the spherical bubble cloud in water is numerically investigated at various frequencies and pressure amplitudes.

The ultrasound pressure wave focuses to the center of the cloud and the pressures inside the bubbles increase extremely when the frequency of the ultrasound is around the 1st mode frequency of the cloud. The maximum pressure inside bubbles reaches 400 MPa and the bubble oscillates with very high frequency in case of 125 kPa pressure amplitude at a frequency, 170 kHz. Additionally, in the case of 100 kPa and 125 kPa pressure amplitudes, the high pressures appear even when the frequency is much lower than the 1st mode frequency estimated by the small amplitude approximation. The peak pressure in the frequency response curves is also seen at 600 kHz, which is about three times as the 1st mode. This is the 2nd mode oscillation of the cloud and its peak pressure is much lower than that in case of the 1st mode.
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REFERENCES


