**ABSTRACT**

Multiple concentrated vortices are often produced in the wake of lifting surfaces and downstream of pumps, turbines, and propulsors. The roll-up of multiple vortex strands is a common feature of these flows. In this study, we are examining the interaction of two vortices of variable strength and relative rotation (e.g. co-rotating or counter-rotating). A pair of equal-strength co-rotating vortices will merge to form a single vortex. However, as the relative strength of the vortices is decreased, the weaker vortex can wrap around the stronger vortex, causing the weaker vortex to be stretched. This stretching process can lead to cavitation inception. In the present work, we will examine this inception process.

**INTRODUCTION**

Multiple concentrated vortices are often produced in the wake of lifting surfaces, and downstream of pumps, turbines, and propulsors. This is particularly true in the tip region of lifting surfaces, as tip-leakage vortices are formed and roll up with vortices formed in the spanwise blade wake. The process of tip vortex roll-up has been extensively examined by many researchers, and recent reviews are found in Green (1995) [1] and Spalart (1998) [2]. The presence of concentrated vortices can lead to discrete vortex cavitation, and Arndt (2003) [3] has recently reviewed this phenomenon.

The inception of vortex cavitation can be predicted and scaled with knowledge of the average vortex properties, typically the vortex strength and viscous core size. The classical scaling of McCormick (1962) [4] is an example. Here, inception is called when the average core pressure drops below vapor pressure. For the case of trailing vortex systems, the location of minimum average core pressure occurs within less than a chord length of the trailing edge, where the shed vorticity has rolled up into a single, strong vortex. A reduction in free-stream pressure combined with a suitable amount of free-stream nuclei will result in cavitation occurring in this region of average minimum pressure.

Recent experiments reported by Chesnakas and Jessup (2003) [5] and Oweis et al. (2003) [6] have demonstrated that inception in the wake of a ducted rotor can occur before the average pressure of the strongest vortex reaches vapor pressure. Instead, limited event rate cavitation occurred farther downstream in the regions of much higher pressure. There are multiple co- and counter-rotating vortices of varying strength in the rotor and, it was hypothesized that inception was occurring as a result of vortex-vortex interactions. It is understood that weak, secondary vortices in wakes and jets are often the first vortices to cavitate (see, for example, Ran and Katz (1994) [7] and Iyer and Ceccio (2002) [8]). Secondary vortices, often oriented in the stream-wise direction, are stretched by stronger, span-wise vortices. This can result in a substantial pressure reduction in the secondary vortices and result in cavitation inception at relatively high pressures.

The interaction of discrete line vortices can also lead to the rapid stretching of secondary vorticity. Co-rotating vortices can orbit and merge in the classical roll-up process. Devenport et al. (1996) [9] and Chen et al. (1999) [10] have shown that the merger of co-rotating vortices can lead to the breakdown of the weaker vortex into fragments during the final stages of merger, at it is expected that these filaments will be stretched in the rotational flow field of the stronger line vortex. Ortega and Savas (2001) [11] and Ortega et al. (2003) [12] have shown that counter-rotating vortices of unequal strength can experience a short-wave instability that leads to the formation of “Ω” like structures on the weaker vortex as it wraps around the stronger line vortex. This process is also accompanied by secondary vortex stretching. We are currently examining how the roll-up process of discrete stream-wise vortices can lead to cavitation inception. We use two hydrofoils to create a pair of vortices with variable strength and relative circulation (i.e. a co-rotating or counter-rotating pair). Presented here are some of our initial observations.

**EXPERIMENTAL SETUP**

Experiments are being conducted in the University of Michigan’s 9-Inch Cavitation Tunnel (Figure 1). The water tunnel has a circular contraction downstream of a series of flow management screens with contraction ratio 6.4:1. The test section has a 22.9 cm diameter round inlet that is then faired into a rectangular test section with widely rounded corners. Four acrylic windows (93.9 cm by 10.0 cm viewing area) permit optical access to the test section flow. The flow in the test section can be operated at pressures from vapor pressure to approximately 200 kPa. The average velocity in the test section...
is variable up to 18 m/s. A de-aeration system can be used to vary the dissolved gas content of the flow, and the inlet water is filtered to 1 microns.

A vortical flow was created using two cambered hydrofoil mounted to two windows of the test section. A schematic diagram is shown in Figure 2, and the top view of the test section is shown in Figure 3. The hydrofoils have a rectangular planform of 9.3 cm span and 16.8 cm chord, and the tip of the hydrofoil was truncated with sharp edges. The gap between the hydrofoil tips was 2.0 cm. The hydrofoil mount allows continuous changes of the incident flow angle. A series of tip and trailing edge vortices will be shed near the tip, and these vortices will merge to form a single vortex within one-half chord length downstream of the hydrofoil trailing edge. The tip vortex produced by the hydrofoil can be visualized with developed cavitation, as shown in Figure 4 for the case of a single installed hydrofoil. Measurements of the vortex interactions were conducted using a free-stream velocity of 12 m/s and a variety of pressures. The dissolved oxygen content was measured with an Orion Model 810 dissolved oxygen meter. In order to reduce the number of free-stream nuclei, the free-stream gas-content was reduced to below 1.5 ppm during the measurements.

Planar Particle Imaging Velocimetry (PIV) was used to measure the vortical flow field at a station 9.0 cm downstream of the trailing edge. A double-pulsed light sheet 2 mm thick was created perpendicular to the mean flow direction using two pulsed Nd:YAG lasers (Spectra Physics model Pro-250 Series). 15-micron average diameter silver coated glass spheres (from Potters Industries) were used to seed the flow. An acrylic prism was optically mounted to a window of the test-section for viewing of the light sheet with reduced optical distortion. The light sheet was imaged with a PIV image capture system produced by LaVision Inc. Double-pulsed images of the light sheet were acquired with a digital camera with 1280 x 1024 pixels. Optical distortion of the planar light sheet image was corrected through a calibration procedure that employed the imaging of a regular grid in the location of the light sheet plane. Velocity vectors were produced from the double-pulsed images using the LaVision image analysis software DaVis 6.0.4. Multi-pass processing with a final window size of 32 x 32 pixels was used with 50% window overlap in the final pass to produce 80 by 52 in plane velocity vectors at 1.1 mm spacing. The velocity field was corrected for the non-parallel orientation of the laser light sheet and the imaging plane through knowledge of the optical geometry and the free-stream velocity. Still images of the cavitating flow were acquired with a 35 mm SLR camera using stroboscopic lighting.

The Reynolds number of the flow based on the freestream velocity and chord length is 2.5x10^6, and the cavitation number is defined by \( \sigma = (P_{\infty} - P_1) / \frac{1}{2} \rho U_{\infty}^2 \), where the free-stream velocity, \( U_{\infty} \), and pressure, \( P_{\infty} \), are measured at the inlet of the test section upstream of the hydrofoils.

**CO-ROTATING VORTEX PAIRS**

The two hydrofoils produce (at least) four distinct vortices: two tip vortices, and two leading edge vortices that form near the upstream, outboard corner of the hydrofoils. The strength and relative circulation of these vortices are a function of the hydrofoil attack angles, \( \alpha_1 \) and \( \alpha_2 \). Looking at the trailing edge of the hydrofoils from downstream, a positive angle of attack leads to a tip vortex with clockwise rotation near the tip of the hydrofoil on the right side, and a clockwise rotating vortex forming downstream of the left-hand hydrofoil. The attack angle is measured from the flat pressure side of the cambered hydrofoil, so a vortex is formed when \( \alpha = 0 \). When \( \alpha_1 \approx \alpha_2 \), two strong co-rotating vortices are formed, and they merge in the expected fashion. Figure 5 shows this merging process visualized by cavitation, for \( \sigma \sim 1.0 \). Inception for this case occurred in the location of vortex merger.

The leading edge vortex is also co-rotating with respect to the stronger leakage vortex. In some cases, this vortex merges with the tip-leakage vortex. But, it is possible that the weaker vortex will be captured in the rotational field of the stronger vortex and be wrapped around it. A close-up view of this process is shown in Figure 6. The pressure has been reduced to visualize the wrapping of the smaller vortices. For the conditions shown in Figure 6, intermittent inception was observed to occur in the stronger vortex before or nearly simultaneously with the inception in the wrapped secondary vortices.

**COUNTER-ROTATING VORTEX PAIRS**

If \( \alpha_1 \approx -\alpha_2 \), two counter rotating vortices of nearly equal strength are produced. These vortices do not interact over the length of the test section. It is expected that the vortices would eventually undergo a long-wave instability as discussed by Crow (1970) [13]. However, the growth rate of this instability is such that it is not detected within the observable region of the test section.

If the strengths of two counter-rotating vortices are significantly mis-matched, cooperative instabilities can occur that results in the wrapping of the weaker vortex around the stronger vortex, forming "\( \Omega \)" type vortex hoops around the stronger vortex. A schematic diagram of this process is shown in Figure 7, after a figure in Ortega and Savas (2001) [11]. It is also possible for a very weak co-rotating vortex to be wrapped around the core of a stronger vortex, as mentioned above, although this process would not necessarily yield the "\( \Omega \)" type loops.

This instability was observed for the pair of two counter-rotating vortices of unequal strength. One hydrofoil was positioned at \( \alpha_1 = -\alpha_2 \), while the attack angle of the second hydrofoil was varied from \( 0^\circ < \alpha_2 < 2^\circ \). This produced a pair of counter-rotating tip vortices of varying strength. Figure 8 shows two images of the vortices (not taken simultaneously) showing the growth and formation of the "\( \Omega \)" type loops.

Figure 9 shows the development of the vortex interactions for varying cavitation numbers and attack angles, \( \alpha_2 \). For small \( \alpha_2 \), the unstable secondary vortex cavitates before the stronger...
primary vortex. However, as the strength of the weaker vortex increases, it no longer undergoes the “Ω”-type looping, and the stronger, unperturbed vortex cavities first when the pressure is lowered. These trends are summarized in Figure 10.

PIV IMAGING OF THE VORTICES

Planar PIV images were taken of the vortex pairs before their merger or unstable interaction. An identification procedure was used to find the best-fit Gaussian vortices to represent the concentrated regions of out-of-plane vorticity computed from the velocity field. Details of the identification process are found in Oweis and Ceccio (2003) [13]. The vortices are characterized by a strength, \( \Gamma \), and core radius, \( a \).

Figure 11 shows the case for two co-rotating vortex pairs (\( \alpha_1 = 3^\circ \), \( \alpha_2 = 4^\circ \)) that merged into a single vortex. Here, \( \Gamma_1 = 0.140 \text{ m}^2/\text{s} \), \( \Gamma_2 = 0.098 \text{ m}^2/\text{s} \), and the core radii are 0.005 m. Figure 12 shows the case of two counter-rotating vortices when the “Ω” type looping did not occur (\( \alpha_1 = 0^\circ \), \( \alpha_2 = -2^\circ \)). In this case, \( \Gamma_1 = 0.096 \text{ m}^2/\text{s} \), \( \Gamma_2 = -0.102 \text{ m}^2/\text{s} \), and the core radii are 0.005 m. Lastly, Figure 13 shows the case of two co-rotating vortices before they undergo the looping instability (\( \alpha_1 = -4^\circ \), \( \alpha_2 = 0^\circ \)). \( \Gamma_1 = -0.154 \text{ m}^2/\text{s} \), \( \Gamma_2 = 0.063 \text{ m}^2/\text{s} \), and the core radii are \( a_1 = 0.006 \text{ m} \) and \( a_2 = 0.004 \text{ m} \). The distances between the vortex centers, \( \delta \), varies between 20 and 25 mm, which is approximately the gap spacing between the hydrofoils.

VORTEX-PAIR INSTABILITIES

The instabilities occurring between line vortices have been examined by a number of researchers, including Crow (1970) [14], Crouch (1997) [15], and Ortega and Savas (2001) [11]. Fabre et al. (2002) [16] analyzed the instability of two counter-rotating vortex pairs, and reported the optimal growth rates for a given vortex spacing, strength ratio, and vortex core sizes. A number of ultra-short, short, and medium wavelength unstable modes were identified. The configuration of the vortices examined here correspond to the case where the trailing pairs are far apart, and the unstable mode corresponds to a modified Crow instability, as discussed by Ortega and Savas (2001) [11]. It is not clear if the wrapping of the weak and strong co-rotating vortices also is in the class of the short-wave instabilities identified by Crouch (1997) [15].

The experimental study of Ortega et al. (2003) [12] identified several conditions of a four-vortex system (i.e. two tip and two flap-generated vortices) where the looping instability occurred. In this study, \(-0.37 < \Gamma_{weak} / \Gamma_{strong} < -0.67\), and the ratio of the vortex separation distance with the vortex core radii was varied between \(3 < \delta / a < 7\). For the unstable case shown in Figure 13, \( \Gamma_2 / \Gamma_1 = -0.41 \) and \( \delta / a = 3.6\), which is consistent with the above range. Moreover, the values of the counter-rotating case from Figure 12 (\( \Gamma_2 / \Gamma_1 = -0.94\)) fall outside the identified range. When the looping instability occurs, its wavelength is on the order of 35 mm (1.5 times the vortex separation distances) and occurs approximately 10 vortex separation distances downstream from the trailing edge. Ortega et al. (2003) [12] found that the wavelength of the looping vortex was approximately 4 times the initial vortex separation. Fabre et al. (2002) [16] has shown that the most unstable modes can be strongly related to the initial vortex parameters.

CONCLUSIONS

We are examining the interaction of vortex interactions that result in cavitation inception. In particular, we are examining how cooperative vortex instabilities can lead to stretching of weak vortices by the stronger vortex. For the case of co-rotating vortices, inception occurs in the region of vortex combination. A weak co-rotating vortex can wrap around a much stronger vortex, but we have not yet observed the independent inception of the weaker vortex in this case. Counter-rotating vortices can undergo a Crow-like instability of relatively short wavelength and amplification. In this case, the stretched weaker vortices will cavitate before the stronger, unperturbed vortex. Moreover, the location of inception can be relatively far downstream from the position of vortex origin. We will continue to investigate the case of weak/strong co-rotating and unstable counter-rotating vortices. Lastly, we will begin to quantify how variable nuclei populations influence the inception of these vortex systems.

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REFERENCES


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**Figure 1:** Photograph of 9-Inch Water Tunnel at the University of Michigan.

**Figure 2:** Schematic diagram of the two-hydrofoil setup.

**Figure 3:** Top view of the test section with two hydrofoils installed on side windows.

**Figure 4:** Side view of the test section with one hydrofoil installed.
Figure 5: Photographs of vortex merging process visualized by cavitation in the case of two strong co-rotating vortices. \((U_\infty = 12\text{m/s}, P_\infty = 75\text{kPa})\)

Figure 6: Photograph of two cavitating tip vortices with leading edge vortex wrapping; (viewing area: 7.5 cm x 10 cm, \(U_\infty = 12\text{m/s}, P_\infty = 75\text{kPa}\))

Figure 7: Schematic diagram of the process forming “Ω” type vortex hoops around the stronger vortex. (Figure adapted from Ortega and Savas, (2001) [11])

Figure 8: Photographs of unstable vortex interaction process visualized by cavitation in the case of two counter-rotating vortices of unequal strength. \((U_\infty = 12\text{m/s}, P_\infty = 75\text{kPa})\)
Figure 9: Photographs of the Interaction of Vortex Cavitation for different pair of vortex (viewing area: 18 cm x 13 cm, distance from the center of view to the trailing edge of the foil is 2.7 chord-lengths, $\alpha_1 = -5^\circ$, $U_\infty = 12$ m/s)

Figure 10: Interaction of Vortex Cavitation for different pair of vortex ($\alpha_1 = -5^\circ$, $U_\infty = 12$ m/s)
Figure 11: Velocity field (mean subtracted) and in-plane vorticity magnitudes $\alpha_1=3^0, \alpha_2=4^0$; the identified vortex properties are $\Gamma_1=0.14 m^2/s$, $\Gamma_2=0.098 m^2/s$, $a_1=0.005 m$, $a_2=0.005 m$, $U_\infty=12 m/s$)

Figure 12: Velocity field (mean subtracted) and in-plane vorticity magnitudes for $\alpha_1=0^0, \alpha_2=-2^0$; the identified vortex properties are $\Gamma_1=0.096 m^2/s$, $\Gamma_2=-0.102 m^2/s$, $a_1=0.005 m$, $a_2=0.005 m$, $U_\infty=12 m/s$)

Figure 13: Velocity field (mean subtracted) and in-plane vorticity magnitudes for $\alpha_1=-4^0, \alpha_2=0^0$; the identified vortex properties are $\Gamma_1=0.154 m^2/s$, $\Gamma_2=0.063 m^2/s$, $a_1=0.006 m$, $a_2=0.004 m$, $U_\infty=12 m/s$)