LINEAR STABILITY ANALYSIS OF THE EFFECTS OF CAMBER AND BLADE THICKNESS ON CAVITATION INSTABILITIES IN INDUCERS

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ABSTRACT

It has been shown by a stability analysis that various cavitation instabilities occur when the cavity length reaches about 65% of the blade spacing [1]. On the other hand, criteria based on pressure gradient near the throat is proposed on experimental basis [2]. The present study is motivated to examine the latter criteria from the viewpoint of stability analysis. For this purpose, steady and stability analyses were carried out for cavitating flow in cascades with circular arc blades and plano-convex blades by a singularity method based on closed cavity model. It was found that the cavitation instability criterion based on the pressure gradient can be adequate as well as the criterion based on the cavity length. It was also found that steady cavity length and the stability of the flow in both cascades can be almost the function of $\sigma(2(\alpha-\alpha_0))$, where σ is a cavitation number, α is an angle of attack and α_0 is a shockless angle of attack.

INTRODUCTION

High-speed turbomachinery, such as turbopumps for rocket engine is operated under cavitating condition. Cavitation can become unstable even at design flow rate and may cause adverse effects on the reliability of the system.

To predict possible cavitation instabilities, steady flow and its stability analyses have been performed by the authors [1,3]. Through a series of the studies, it was found that various cavitation instabilities can occur when the steady cavity length reaches about 65% of the blade spacing of impeller. Actually, such phenomenon has been observed in some experiments [4,5].

On the other hand, it has been suggested on experimental basis that the cavitation instabilities can occur when pressure gradient near the throat becomes smaller [2]. To examine it on the standpoint of theoretical analysis, the steady flow and its stability analyses are carried out for cavitating flow in cascades

with circular arc blades and plano-convex blades by a singularity method based on closed cavity model. The effects of camber and blade thickness are discussed for the steady and unsteady cavitation. And also the relation between the pressure gradient on the suction surface of a blade near the throat and the occurrence of cavitation instabilities is investigated.

NOMENCLATURES

A	= coefficient matrix
B	= constant vector
C	= chord length
C_p	= pressure coefficient, $(p - p_{-Ls})/(\rho U^2)$
f	= function used in equation of complex velocity
h	= pitch
i	= imaginary unit in space, $i^2 = -1$
j	= imaginary unit in time, $j^2 = -1$
L	= distance between the leading edge of blade and the space with constant total pressure, $100C$
l	= cavity length
N	= amplitude of upstream axial velocity fluctuation
N_C, N_B	= number of discrete points on s -coordinate
p_L	= pressure at $\xi = -L$
p_t	= total pressure at $\xi = -L$
p_v	= vapor pressure
q	= strength of source distribution
q_B	= strength of source distribution which represents blade thickness
S	= location of discrete point on s -coordinate
St	= Strouhal number, $(\omega_R/(2\pi))/(U/l_s)$
t	= maximum camber, or maximum length between suction surface and chord, or time
U	= upstream mean velocity
U_T	= peripheral velocity (moving speed) of cascade, $U \sin(\alpha + \beta)$

$$\begin{aligned}
&= U e^{-i\alpha} + \tilde{N} e^{i\beta} e^{j\omega t} \\
&+ \frac{1}{2\pi} \sum_{n=0}^{Z_N-1} \left\{ \left[\int_0^1 [q_n(s_1) + i\gamma_{1,n}(s_1) + q_{B,n}(s_1)] \right. \right. \\
&\times [f_n(z, l_n s_1) - f_n(-L + a e^{i(\pi/2-\beta)}, l_n s_1)] l_n ds_1 \\
&+ \left. \int_1^2 [i\gamma_{2,n}(s_2) + q_{B,n}(s_2)] \right. \\
&\times [f_n\{z, (C-l_n)s_2 + (2l_n-C)\} \\
&- f_n\{-L + a e^{i(\pi/2-\beta)}, (C-l_n)s_2 + (2l_n-C)\}] \\
&\times (C-l_n) ds_2 \\
&+ \left. \left. \int_C^\infty \gamma_{t,n}(\xi) [f_n(z, \xi) - f_n(-L + a e^{i(\pi/2-\beta)}, \xi)] d\xi \right\} \right\} \\
&\quad (3)
\end{aligned}$$

$$\begin{aligned}
f_n(z, \xi) &= \frac{\pi}{Z_N h} e^{-i(\pi/2-\beta)} \\
&\cot \left\{ \frac{\pi}{Z_N h} (z - \xi) e^{-i(\pi/2-\beta)} - \frac{n}{Z_N} \pi \right\} \\
&\quad (4)
\end{aligned}$$

where s_1 and s_2 are introduced so that the intervals of integrals in Eq.(3) become constant in spite of the variable cavity length l_n , and defined by

$$\xi = l_n s_1 \quad (0 \leq \xi \leq l_n, 0 \leq s_1 \leq 1, n=0, 1, \dots, Z_N-1) \quad (5)$$

$$\begin{aligned} \xi &= (C-l_n)s_2 + (2l_n-C) \\ (l_n \leq \xi \leq C, 1 \leq s_2 \leq 2, n=0, 1, \dots, Z_N-1) \end{aligned} \quad (6)$$

The function $f_n(z, \xi)$ represents the effects of infinite number of singularities of equal strength located at $z = \xi + nZ_N h e^{i(\pi/2-\beta)}$, $n = -\infty, \dots, -1, 0, 1, \dots, \infty$. The strength of singularities is specified on these coordinates s_1 and s_2 moving in accord with the variable cavity length l_n . In the present analysis with larger $L (=100C)$, $f(-L + a e^{i(\pi/2-\beta)}, \xi)$ is approximated to $-\pi e^{i\beta}/h$, where a is an arbitrary constant.

The strength of singularities and the cavity length are divided into the steady and unsteady components as follows.

$$\begin{aligned}
q_n(s_1) &= q_{s,n}(s_1) + \tilde{q}_n(s_1) e^{j\omega t} \\
\gamma_{1,n}(s_1) &= \gamma_{1s,n}(s_1) + \tilde{\gamma}_{1,n}(s_1) e^{j\omega t} \\
\gamma_{2,n}(s_2) &= \gamma_{2s,n}(s_2) + \tilde{\gamma}_{2,n}(s_2) e^{j\omega t} \\
q_{B,n}(s_1) &= q_{Bs,n}(s_1) \\
q_{B,n}(s_2) &= q_{Bs,n}(s_2) \\
\gamma_{t,n}(\xi) &= \tilde{\gamma}_{t,n}(\xi) e^{j\omega t} \\
l_n &= l_{s,n} + l_n e^{j\omega t} \\
(n=0, \dots, Z_N-1) &\quad (7)
\end{aligned}$$

where $\omega = \omega_R + j\omega_I$ is the complex frequency with its real part ω_R signifying the frequency and the imaginary part ω_I the decay rate.

It is assumed that the outlet duct length is infinite and no velocity fluctuation exists at downstream infinity. The inlet duct length L is assumed to be finite and set to be $100C$ in the present study. The inlet duct is connected to a space with constant total pressure p_t along AB at the duct inlet. The complex conjugate velocity fluctuation there is denoted by $\tilde{N} e^{i\beta} e^{j\omega t}$, where $\tilde{N}$ represents the amplitude of axial velocity fluctuation.

The velocity around the cascade is divided into the uniform

steady component ($U, U\alpha$), the steady disturbance (u_s, v_s) and the unsteady disturbance ($\tilde{u}, \tilde{v}$) components as follows.

$$\begin{aligned}
u &= U + u_s + \tilde{u} e^{j\omega t} \\
v &= U\alpha + v_s + \tilde{v} e^{j\omega t}
\end{aligned} \quad (8)$$

Here, it is assumed that $\alpha \ll 1$, $U \gg |u_s|, |v_s| \gg |\tilde{u}|, |\tilde{v}|$ and linearization is made on these assumptions in the present study.

BOUNDARY CONDITIONS

Boundary conditions are applied on the coordinates s_1 and s_2 in accord with the variable cavity length, l_n .

Boundary Condition on Cavity Surface. The pressure on the cavity surface ($z = m h e^{i(\pi/2-\beta)} + l_m s_1 + 0i$, $0 \leq s_1 \leq 1$, $m=0, 1, \dots, Z_N-1$) is assumed to be constant and equal to the vapor pressure p_v . By applying the linearized momentum equation on the cavity surface, we obtain

$$\frac{\partial u_{c,m}}{\partial t} + U \frac{\partial u_{c,m}}{\partial \xi} = 0 \quad (9)$$

By solving Eq.(9) in consideration of Eq.(5), the velocity $u_{c,m}(s_1)$ on the cavity surface is given by

$$u_{c,m}(s_1) = U + u_{cs} + \tilde{u}_{c,m} e^{-j \frac{\omega l_{s,m}}{U} s_1} e^{j\omega t} \quad (10)$$

Equations (3) and (10) lead to the following boundary condition.

$$\begin{aligned}
&\text{Real} \left[w(m h e^{i(\pi/2-\beta)} + l_m s_1 + 0i, t) \right] \\
&= U + u_{cs} + \tilde{u}_{c,m} e^{-j \frac{\omega l_{s,m}}{U} s_1} e^{j\omega t}, \quad 0 \leq s_1 \leq 1 \quad (11)
\end{aligned}$$

The velocity on the cavity surface is related with the pressure at $\xi = -L$ as follows, by the unsteady Bernoulli's equation between $\xi = 0$ and $\xi = -\infty$ and the momentum equation for the fluid between $\xi = -L$ and $\xi = -\infty$, which oscillates uniformly with the amplitude $\tilde{N}$.

$$u_{cs} = \frac{p_{-Ls} - p_v}{\rho U} = \frac{\sigma U}{2} \quad (12)$$

$$\begin{aligned}
&\frac{\partial [\Phi_{0,m} - \Phi_{-\infty} - \lim_{a \rightarrow \infty} \{(a - L \cos \beta) \tilde{N}\}]}{\partial t} \\
&+ U \tilde{u}_{c,m} - U \tilde{N} \cos \beta = \frac{\tilde{p}_{-L}}{\rho}
\end{aligned} \quad (13)$$

where p_{-Ls} and p_v are the pressure at the upstream boundary, $\xi = -L$, and the vapor pressure respectively. σ is called the cavitation number and defined as $2(p_{-Ls} - p_v)/(\rho U^2)$. $\Phi_{0,m}$ and $\Phi_{-\infty}$ are the velocity potentials at the leading edge of m th blade and $\xi = -\infty$ respectively. By the assumption that the flow around the cascade is connected to a space with a constant total pressure along AB at a distance L from the leading edge, the following equation can be obtained.

$$\tilde{p}_{-L} = -\rho U \tilde{N} \cos \beta \quad (14)$$

Equations (12) and (13) with Eq.(14) give the tangential velocity u_{cs} and $\tilde{u}_{c,m}$ on the cavity surface.

Boundary Condition on Blade Surface. The y -components of velocity on the blade surface should satisfy the following equations.

For the cascade with circular arc blades,

$$- \text{Im ag}[w\{mhe^{i(\pi/2-\beta)} + l_m s_1 - 0i, t\}] = \frac{U\Omega}{2} \left(1 - 2 \frac{l_m s_1}{C} \right) \quad (0 \leq s_1 \leq 1) \quad (15)$$

$$- \text{Im ag}[w\{mhe^{i(\pi/2-\beta)} + (C - l_m)s_2 + (2l_m - C) \pm 0i, t\}] = \frac{U\Omega}{2} \left(1 - 2 \frac{(C - l_m)s_2 + (2l_m - C)}{C} \right) \quad (1 \leq s_2 \leq 2) \quad (16)$$

For the cascade with plano-convex blades,

$$- \text{Im ag}[w\{mhe^{i(\pi/2-\beta)} + l_m s_1 - 0i, t\}] = 0 \quad (0 \leq s_1 \leq 1) \quad (17)$$

$$- \text{Im ag}[w\{mhe^{i(\pi/2-\beta)} + l_m s_1 + 0i, t\}] = \frac{U\Omega}{2} \left(1 - 2 \frac{l_m s_1}{C} \right) \quad (0 \leq s_1 \leq 1) \quad (18)$$

$$- \text{Im ag}[w\{mhe^{i(\pi/2-\beta)} + (C_m - l_m)s_2 + (2l_m - C_m) - 0i, t\}] = 0 \quad (1 \leq s_2 \leq 2) \quad (19)$$

$$- \text{Im ag}[w\{mhe^{i(\pi/2-\beta)} + (C - l_m)s_2 + (2l_m - C) + 0i, t\}] = \frac{U\Omega}{2} \left(1 - 2 \frac{(C - l_m)s_2 + (2l_m - C)}{C} \right) \quad (1 \leq s_2 \leq 2) \quad (20)$$

Cavity Closure Condition. The cavity thickness η_m should satisfy the following kinematic condition.

$$\frac{\partial \eta_m}{\partial t} + \frac{1}{l_m} \left(U - s_1 \frac{dl_m}{dt} \right) \frac{\partial \eta_m}{\partial s_1} = q_m(s_1) \quad (0 \leq s_1 \leq 1) \quad (21)$$

The cavity thickness is separated into steady and unsteady components as follows.

$$\eta_m(s_1) = \eta_{s,m}(s_1) + \tilde{\eta}_m(s_1)e^{j\omega t} \quad (22)$$

Equation (21) with Eqs.(7) and (22) is separated into steady and unsteady components after linearization. By integrating these equations, we obtain

$$\eta_{s,m}(s_1) = \frac{l_{s,m}}{U} \int_0^{s_1} q_{s,m}(s_1') ds_1' \quad (23)$$

$$\begin{aligned} \tilde{\eta}_m(s_1) &= \frac{l_{s,m}}{U} \int_0^{s_1} \tilde{q}_m(s_1') e^{-j \frac{\omega l_{s,m}}{U} (s_1 - s_1')} ds_1' \\ &+ \frac{\tilde{l}_m}{U} \int_0^{s_1} \left(1 + j \frac{\omega l_{s,m}}{U} s_1' \right) q_{s,m}(s_1') e^{-j \frac{\omega l_{s,m}}{U} (s_1 - s_1')} ds_1' \end{aligned} \quad (24)$$

The cavity closure conditions for steady and unsteady components are as follows.

$$\eta_{s,m}(1) = 0 \quad (25)$$

$$\tilde{\eta}_m(1) = 0 \quad (26)$$

Kutta's Condition. The pressure difference across the blade is assumed to vanish at the trailing edge. Making subtraction of momentum equations on the pressure and suction surface of the blade after linearization, the above condition can be represented by the following equation.

$$\begin{aligned} \frac{d}{dt} \left[\int_0^1 \gamma_{1,m}(s_1) l_m ds_1 + \int_1^2 \gamma_{2,m}(s_2) (C - l_m) ds_2 \right] \\ + U \gamma_{2,m}(2) = 0 \end{aligned} \quad (27)$$

Equation (27) is equivalent to Kelvin's circulation conservation law and signifies that the vortex wake of strength $\gamma_{2,m}(2)$ is shed from the trailing edge with the mean velocity U , corresponding to the change of the blade circulation.

For the case that the trailing free vortex of the strength $\tilde{\gamma}_{t,m}(\xi)$ is transported on the free stream, it can be related with $\tilde{\gamma}_{2,m}(2)$ as follows.

$$\tilde{\gamma}_{t,m}(\xi) = \tilde{\gamma}_{2,m}(2) e^{-j \frac{\omega l_{s,m}}{U} (\xi - C) / l_{s,m}} \quad (28)$$

Downstream Condition. In the case that the downstream duct length is assumed to be infinite, the downstream flow rate fluctuation is suppressed due to the infinite inertia effect. Then, the continuity equation can be given by

$$\tilde{N} + \frac{1}{Z_N h} \sum_{n=0}^{Z_N-1} \left\{ \int_0^1 \tilde{q}_n(s_1) l_n ds_1 \right\} = 0 \quad (29)$$

Closure Condition for $q_{B,m}$. The unknowns $q_{B,m}$ which represents the blade thickness should satisfy the following equation.

$$\int_0^1 q_{B,s,m}(s_1) l_m ds_1 + \int_1^2 q_{B,s,m}(s_2) (C - l_m) ds_2 = 0 \quad (30)$$

PROCEDURE OF CALCULATION

The unknowns are the steady and unsteady parts of strength of source and vortexes, $q_m(s_1)$, $q_{B,s,m}(s_1)$, $q_{B,s,m}(s_2)$, $\gamma_{1,m}(s_1)$, $\gamma_{2,m}(s_2)$, cavity length l_m , velocity $u_{c,m}$ on cavity surface, and the amplitude of inlet velocity fluctuation $\tilde{N}$. These are determined by the Eqs.(11)-(20), (23)-(30). The strength of singularities are specified at discrete points, $s = S_{1,k} (k=1 \sim N_C)$, on cavity surface and $S_{2,k} (k=1 \sim N_B)$ on wetted surface in the coordinates stretching with the variable cavity length l_m as unknowns. In the present study, N_C and N_B are set to be 37. The boundary conditions are applied at the middle of the discrete points. After linearization, the boundary conditions are reduced to the following set of linear equations.

For steady components,

$$A_s(l_{s,m}) \begin{Bmatrix} q_{s,m}(S_{1,1}) \\ \vdots \\ \gamma_{1s,m}(S_{1,1}) \\ \vdots \\ \gamma_{2s,m}(S_{2,1}) \\ \vdots \\ q_{B,s,m}(S_{1,1}) \\ \vdots \\ q_{B,s,m}(S_{2,1}) \\ \vdots \\ \sigma \end{Bmatrix} = B_s \quad (31)$$

($m=0, 1, \dots, Z_N-1$)

and for unsteady components,

Fig.3 Steady cavity length in the cascade with circular arc blades and plano-convex blades. $C/h=2$, $\beta=80^\circ$. $\alpha=5^\circ$. α_0 is a shockless angle of attack for non-cavitating flow.

$$A_u(l_{s,m}, \omega) \begin{Bmatrix} \tilde{q}_m(S_{1,1}) \\ \vdots \\ \tilde{\gamma}_{1,m}(S_{1,1}) \\ \vdots \\ \tilde{\gamma}_{2,m}(S_{2,1}) \\ \vdots \\ \tilde{u}_{c,m} \\ \vdots \\ \tilde{l}_m \\ \vdots \\ \tilde{N} \end{Bmatrix} = \begin{Bmatrix} 0 \\ \vdots \\ 0 \end{Bmatrix} \quad (32)$$

$(m=0, 1, \dots, Z_N-1)$,

where $A_s(l_{s,m})$ and $A_u(l_{s,m}, \omega)$ are coefficient matrices, B_s is a constant vector. The values of unknowns on m th blade are shown by the index $m(=0, 1, \dots, Z_N-1)$.

The steady flow is determined by Eq.(31). Equation (32) is a set of linear homogeneous equations. The determinant of the coefficient matrix $A_u(l_{s,m}, \omega)$ should satisfy

$$|A_u(l_{s,m}, \omega)|=0 \quad (33)$$

so that non-trivial solutions can be obtained. The complex frequency $\omega=\omega_R+j\omega_I$ is determined from Eq.(33). Thus, the angular frequency ω_R and the stability of cavitation depend on the steady cavity length $l_{s,m}$ or equivalently σ .

RESULTS AND DISCUSSIONS

Steady Cavity Length. Figure 3 shows steady cavity length in both cascades with circular arc blades and plano-convex blades. The solidity C/h is 2, the stagger angle β is 80° and the number of blades Z_N is 3. The values of l_s/h are plotted against $\sigma/(2\alpha)$ in Fig.3(a). It is found that the steady cavity length is smaller for larger camber and blade thickness and the difference of the effects of the camber and blade thickness is small for the steady cavity length. Symbols in Fig.3(a) show the onset point of the rotating cavitation obtained by the stability analysis. The value of $\sigma/(2\alpha)$ at the onset of the rotating cavitation is smaller for larger camber and blade thickness. The

Fig.4 Destabilizing roots for the flat plate cascade with $C/h=2$ and $\beta=80^\circ$.

rotating cavitation occurs for the steady cavity length longer than the 65% of blade spacing.

The steady cavity length l_s/h plotted against $\sigma/(2(\alpha-\alpha_0))$ is shown in Fig.3(b) and (c) for the cascades with circular arc blades and plano-convex blades respectively. α_0 is a shockless angle of attack for non-cavitating flow and the values of α_0 are shown in each figure. From these results, the steady cavity length can be practically correlated with $\sigma/(2(\alpha-\alpha_0))$ for both cascades.

Stability Analysis. Figures 4-6 show the Strouhal number St of unstable modes and the phase difference $\theta_{n,n+1}$ for the cascades with flat plate blades, circular arc blades and plano-convex blades respectively. Strouhal number in the present study is based on steady cavity length l_s and defined as follows.

(a) $t/C=0.01$ ($\alpha_0=1.425^\circ$)

(b) $t/C=0.02$ ($\alpha_0=2.846^\circ$)

Fig.5 Destabilizing roots for the cascade with circular arc blades. $C/h=2$ and $\beta=80^\circ$. $\alpha=5^\circ$.

(a) $t/C=0.01$ ($\alpha_0=1.299^\circ$)

(b) $t/C=0.02$ ($\alpha_0=2.598^\circ$)

Fig.6 Destabilizing roots for the cascade with plano-convex blades. $C/h=2$ and $\beta=80^\circ$. $\alpha=5^\circ$.

$$St = \frac{\omega_R / 2\pi}{U / l_s} \quad (34)$$

Many unstable modes are obtained but only lower order modes are shown in Figs.4-6.

Mode I and V are the cavitation surge modes with same phase for all blades. Mode I and V are the modes whose Strouhal numbers are dependent and independent on the length L of the inlet duct respectively.

Mode II-IV correspond to the modes of rotating cavitation. Mode II represents the forward rotating cavitation, Mode III and IV represent the backward rotating cavitation and higher order mode of forward rotating cavitation respectively.

Figure 5(a) and (b) show the results on the cascades with circular arc blades whose cambers t/C equal 0.01 and 0.02 respectively. From Fig.4, Fig.5(a) and (b), it is found that, as the value of t/C increases, the values of $\sigma/(2(\alpha-\alpha_0))$ at the onset of Mode II-IV corresponding to the rotating cavitation decrease and those of Mode I and V corresponding to the cavitation surge increase.

Figure 6(a) and (b) show the results on the cascades with plano-convex blades whose thicknesses t/C equal 0.01 and 0.02 respectively. As the value of t/C increases, the values of $\sigma/(2(\alpha-\alpha_0))$ at the onset of Mode II-IV decrease and those of Mode I

Fig.7 Steady pressure distribution on the suction surface of a blade in the cascades with flat blade ($t/C=0$) and circular arc blades. $C/h=2$, $\beta=80^\circ$. $\alpha=5^\circ$.

and V increase as well as the case of the cascades with circular arc blades.

As shown in Fig.4-6, many various unstable modes appear for the steady cavity length longer than the 65% of blade spacing. This means that the steady cavitation with longer cavity over the 65% of blade spacing is unstable in spite of the camber and blade thickness. Therefore, the rule that the various cavitation instabilities occur in the case with the cavity longer than the 65% of the blade spacing is adequate also for the cascades with camber and thickness.

Steady Pressure Distribution and Pressure Gradient on Blade Figures 7 and 8 show the steady pressure distributions on the circular arc and plano-convex blades respectively, at the onset of the rotating cavitation. Fig.7(a) shows the pressure distribution on the flat blade cascade. The regions with constant pressure in Figs.7 and 8 correspond to cavities. The pressure coefficient C_p tends to 0.5 if we approach the cavity trailing edge from downstream. This is caused by the flow stagnation due to the linear closed cavity model.

Here, we consider the pressure gradient at the throat $\xi/h=\sin\beta$ where the effect of the flow stagnation at the cavity trailing edge is insignificant. Dot-dashed lines at $\xi/h=\sin\beta=0.985$ in Figs.7 and 8 represent the location of the throat.

Figure 9 shows the steady pressure gradients on the suction surface of blades at the throat, $\xi/h=0.985$. Figs. 9(a) and (b) are for the cascades of circular arc blades and plano-convex blades

Fig.8 The same as Fig.7, for the cascades with plano-convex blades.

respectively. As the value of $\sigma/(2(\alpha-\alpha_0))$ decreases, the pressure gradient $d(C_{ps})/d(x/C)$ decreases. The negative slope is caused by the flow stagnation near the cavity trailing edge. The sharp increase near $\sigma/(2(\alpha-\alpha_0))=0.65-0.75$ corresponds to the shift between cavity surface and the stagnation point.

The symbol (○) in Fig.9(a) and (b) shows the onset point of the rotating cavitation corresponding to ModeII. The rotating cavitation occurs when the pressure gradient becomes small to some degree. This fact suggests that the pressure gradient on the suction surface near the throat can also be used as a criterion for the occurrence of cavitation instabilities. However, the value of critical pressure gradient changes largely for plano-convex blades.

CONCLUSIONS

The analysis of steady flow and the stability analysis were carried out to investigate the effects of the camber and blade thickness, and the relation between the pressure gradient on the suction surface of a blade near the throat and the occurrence of cavitation instabilities.

In the analysis of the steady flow, it was found that the steady cavity length is shorter for the larger camber and blade thickness and the difference of the effects of the camber and blade thickness is small for the steady cavity length. And also it was found that the steady cavity length can be practically correlated with $\sigma/(2(\alpha-\alpha_0))$ for the cascades with circular arc blades and plano convex blades, where α_0 is a shockless angle of attack.

In the stability analysis, various modes appear for the longer cavity over the 65% of the blade spacing in both cascades with circular arc blades and plano-convex blades. From this fact, the criterion based on the steady cavity length is considered to be adequate for the occurrence of the cavitation instabilities for both cascades as well as for flat blade cascade.

The steady pressure gradient on the suction surface of a blade near the throat was calculated. When the pressure gradient

(a) Cascade with circular arc blades

(b) Cascade with plano-convex blades

Fig.9 Pressure gradient on the suction surface of a blade at $\xi=h\sin\beta$. $C/h=2$, $\beta=80^\circ$. $\alpha=5^\circ$.

became small to some degree, the rotating cavitation occurred. Therefore, it is thought that the pressure gradient on the suction surface near the throat can be used as a criterion for the occurrence of cavitation instabilities.

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