COMBINED METHOD OF BODY STABILIZATION IN SHEAR FLOW

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ABSTRACT
The peculiarities of coherent vortical structures that are formed in a shear flow around a backward facing step are considered. The model of this flow is investigated both experimentally and theoretically for laminar and turbulent regimes of flow. The essence of a method of receptivity - interaction of coherent vortical structures is described. This method is considered as a basis for different variants of practical tasks realization.

INTRODUCTION
In various kinds of shear flows there are zones with high concentration of shear stresses. The places near edges of different ledges are an example of such zones. For back edge of facing step the flow separation zone is formed that with growth of speed leads to cavitational streamline of a body. On this edge the pressure jump is formed that initiates disturbances after this edge on the boundary between separating zone and main flow. In [1,2] the coherent vortical structures (CVS) that are formed behind the cavitators of different geometries are investigated experimentally. In [1] the disturbances in the form of waves have been found with the use of high-speed filming. Their parameters were drawn on the Tollmmin-Shlichting stability diagram of waves. In [3] the low-frequency quasilinear fluctuation near to a cavity surface have been also measured. The development CVS behind cavitator in the zone of high concentration of shear stresses on the boundary of a cavity actually occurs in the same order, as on the model breadboard of distributions development in a boundary layer, presented in [4].

NOMENCLATURE
$U_\infty$ - free flow velocity;
$u$, $v$ - longitudinal and normal velocity components;
$x$, $y$ - Cartesian coordinates;
$n$ - normal direction to streamlined surface;
$\rho$ - density;
$\beta$ = $2\pi n$ - circular frequency of fluctuations;
$C_r$ - speed of fluctuations spreading;
$\alpha$ = $2\pi \lambda$ - wave number of revolting fluctuation;
$\lambda$ - wave length of a fluctuation;
$\delta^*$ - displacement thickness;
$\nu$ - kinematical viscosity coefficient;
$\nu_t$ - kinematical coefficient of turbulent viscosity;
$\nu_{eff} = \nu + \nu_t$ - effective viscosity;
$Sc$, $Sc_t$ - molecular and turbulent Schmidt’s numbers;
$\Gamma_u = \Gamma_v = \frac{\nu_{eff}}{u\nu} - \text{dimensionless diffusive factor for velocities};$
$\Gamma_c = \frac{\nu}{Sc} - \text{dimensionless diffusive factor for concentration};$
$Re^*$ - Reynolds number calculated on $\delta^*$;
$c$ - scalar addition concentration;
$c_o$ - initial scalar addition concentration;
$c/c_o$ - dimensionless scalar addition concentration;
$\varphi = \{u, v, c\} - \text{generalized label of a variable};$
$A_x, A_y$ - convective coefficients along $x$ and $y$ respectively;
$F$ - diffusion factor;
$S_o$ - source term;
$L$ - typical length scale;
$\tilde{t} = t \frac{u_\infty}{L} - \text{non-dimensional time};$
$\tilde{p} = \frac{p}{\rho u_\infty^2} - \text{non-dimensional pressure};$
$\tilde{x} = \frac{x}{L}, \tilde{y} = \frac{y}{L} - \text{dimensionless coordinates};$
$\tilde{u} = \frac{u}{u_\infty}, \tilde{v} = \frac{v}{u_\infty} - \text{dimensionless velocity components};$
$\chi$ - switcher for flow type: flat ($\chi = 0$) or axial ($\chi = 1$);
$\theta$ - Speed of towage of model;
$g$ - gravity acceleration;
$V$ - displacement volume of a vessel in a condition of rest;
$R$ - model towing resistance;
$\Delta$ - model weight displacement;
1. MODELING OF DISTURBANCES DEVELOPMENT IN THE FLOW AROUND A FACING STEP

A gas-vapor mixture of a cavity can be presented as an elastic surface with a high pliability, and a layer of high-speed boundary of a liquid contiguous to gas-vapor mixture, - as a thin elastic film. The experimental researches of hydrodynamic stability at such representation have been carried out in the hydrodynamic stand with a small turbulence level with the use of a tellurium-method [5]. The device of the hydrodynamic stand, technique of realization of measurements are given in [5,6]. In a working site of the stand of length 3 м the various kinds of bottom were established. At modeling current behind ledge the design of the bottom was executed as a metal frame fitted outside of thin polyvinyl film. There was an opportunity to adjust height of ledge under a film, its elasticity and tension. In experiments under film there was a water (series of experiences 1-15,21-42), air (series of experiences 16-20), and heated up water (series of experiences 43,44).

By the classical method the hydrodynamic stability was experimentally investigated, all parameters of revolving movement are investigated too. In the table the mechanical characteristics of a film are given at realization of experiences. As it is visible from a fig. 1(a), the area of instability essentially has increased, and the Reynolds number of loss of stability thus has decreased. It was revealed, that the area of instability depends on thickness of a layer of water \( h \), taking place under a membrane. At smaller thickness \( h \) the area of instability was less. At the same time dimensionless wave number and phase speed to a lesser degree depend from \( h \), though the laws are kept same (fig. 1, b). Thus the maximal amounts of these sizes have appeared above than appropriate sizes at a flow of a rigid wall and are displaced in the party of the large Reynolds numbers. The ambiguity of conformity of the maximal amounts of the specified sizes to Reynolds numbers is found out. To deterioration of stability testifies and appreciably increased phase speed and diagrams of dependences of amplitudes of cross speeds of revolving movement from frequency of fluctuation. In a series of experiences 16-20 under a membrane there was air by thickness 2см, and under air there was a layer of water by thickness 5см. Reduction of a tension in a membrane and presence of air have resulted in increase of stability and reduction of a range of unstable fluctuations.

The given results have shown, that the parameters of

<table>
<thead>
<tr>
<th>Numbers of experiments</th>
<th>( E \cdot 10^{-2} ), N/m²</th>
<th>( T ), N/m²</th>
<th>( \rho ), kg/m³</th>
<th>( \eta \cdot 10^{-4} ), m</th>
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</thead>
<tbody>
<tr>
<td>B1-B5</td>
<td>1,92</td>
<td>79,0</td>
<td>950</td>
<td>1</td>
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<tr>
<td>B6-B10, B11-B15</td>
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<tr>
<td>B16-B20</td>
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<td>36,0</td>
<td>950</td>
<td>1</td>
</tr>
<tr>
<td>B21-B25, B26-B34, B27-B35</td>
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<td>17,5</td>
<td>950</td>
<td>1</td>
</tr>
<tr>
<td>B36-B42, BT43-BT44</td>
<td>1,35</td>
<td>25,0</td>
<td>950</td>
<td>1</td>
</tr>
</tbody>
</table>

Fig.1. Neutral curves in coordinates of dimensionless frequency (a), wave number (b) and phase speed (v) At a longitudinal flow of a membrane surface: a - measurement on rigid (1) and membrane surfaces at \( h = 7 \) cm (curve 2, series of experiments 1-5), \( h = 1 \) cm (curve 3, series of experiments 6-10), \( h = 2 \)cm (curve 4, series of experiments 16-20), \( h = 1 \)cm (curve 5, series of experiments 11-20); 6, v - Babenko measurement [5] (1) and Shubayer & Scramsted (2 on a rigid plate), measurement on membrane surfaces [10]: 3 - experiments 1-5, 4 - experiments 6-10, 5 - experiments 16 - 20, 6 - experiments 26-34, 7 - experiments 36-42, 8 - experiments 43-44, 9 - experiments 21-25.
revolting movement essentially depend on speed of the basic flow, ledge size, tension of a film simulating speed of a cavitating flow, properties of a liquid above ledge and under it. As well as in Brennen experiences [4], the frequency of unstable fluctuations essentially has increased in comparison with the classical data at a flow of a rigid plate. The other characteristics of the disturbed motion have been changed too, that results in the accelerated development CVS on the boundary of a shift layer gas - liquid.

2. A RECEPIVITY METHOD OF DIFFERENT DISTURBANCES
The movement of liquid media behind the cavitator causes the appropriate motion of gas in a cavity. Thus it is possible to present the motion of gas-vapor mixture near boundary with a liquid as though along a running wall. Thus, on the liquid boundary - pairs as though cooperate a boundary layer of a liquid and boundary layer in gas-vapor mixture. The fluctuations in these boundary layers will interact among themselves.

In [7] the interaction of various disturbances in a boundary layer is investigated experimentally. The results of research of a problem of receptivity have allowed making two basic conclusions. First, the process of disturbances interaction in a boundary layer can be presented as a superposition of "frozen", so-called natural structures of revolting motion at each stage of transition (at "absence" of disturbances) with structures of external disturbances. Secondly, the disturbances interaction carries resonant character depending on a kind, of energy and the peak-time-wave characteristics of these structures. The mentioned above concerns not only to transitive, but also to turbulent boundary layer, and to other types of disturbances interaction too.

3. THEORETICAL MODELLING OF SEPARATED FLOW AROUND LEDGE
PRINCIPAL STATEMENT OF PROBLEM. The object of this part of research is connected with shear flows spreading around several types of obstacles of streamlined surface geometry. One of them is a flow around facing step that is a part of problem discussed above. The goal of this unit is to consider this kind of flows on the base of mathematical modeling technique, construct the corresponding mathematical model with the use of the Reynolds-averaged Navier-Stokes system of equations (or, as a particular case, its parabolic approximation) and evaluate the possibility of turbulence models to describe correctly the diffusive processes in these flows. The several types of basic two-dimensional shear flows have been investigated. They are presented by the table below.
GOVERNING EQUATIONS. For two-dimensional flow (flat or axisymmetric) the corresponding system of governing equations will have the form
\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0; \quad (1)
\]
\[
\frac{1}{y^2} \left[ \frac{\partial}{\partial x} \left( \Gamma_u y^2 \frac{\partial u}{\partial x} \right) + \frac{\partial}{\partial y} \left( \Gamma_u y^2 \frac{\partial u}{\partial y} \right) - \frac{\partial p}{\partial x} \right] + S_u = 0; \quad (2)
\]
\[
\frac{1}{y^2} \left[ \frac{\partial}{\partial x} \left( \Gamma_v y^2 \frac{\partial v}{\partial x} \right) + \frac{\partial}{\partial y} \left( \Gamma_v y^2 \frac{\partial v}{\partial y} \right) - \frac{\partial p}{\partial y} \right] + S_v = 0; \quad (3)
\]
\[
\frac{1}{y^2} \left[ \frac{\partial}{\partial x} \left( \Gamma \phi \frac{\partial \phi}{\partial x} \right) + \frac{\partial}{\partial y} \left( \Gamma \phi \frac{\partial \phi}{\partial y} \right) - \frac{\partial \phi}{\partial x} \right] + S_c = 0; \quad (4)
\]
where (1) is an equation of continuity; (2, 3) – the momentum equations for a longitudinal \(u\) and normal \(v\) components of velocity; the equation (4) describes the concentration of scalar additive \(c\) that is transferred by the flow. As a first approximation, in the given research the values of Schmidt’s numbers \(S_c, S_v\), were taken as constants. Symbols \(S\) in (2-4) denote the source terms, whose determination depends on problem that is under consideration (for example, an interaction between flow phases).

The system (1-4) is solved under the following boundary conditions,

Streamlined surface:
\[
\bar{u} = 0, \quad \bar{v} = 0, \quad \frac{\partial \phi}{\partial y} = 0; \quad (5)
\]

Initial cross-section \((\bar{x} = \bar{x}_o)\):
\[
\bar{u} = f(\bar{y}), \quad \bar{v} = 0, \quad \bar{\phi} = \phi(\bar{y}); \quad (6)
\]

Output boundaries of computational domain:
\[
\frac{\partial \bar{u}}{\partial n} \to 0; \quad \frac{\partial \bar{v}}{\partial n} \to 0; \quad \frac{\partial \bar{\phi}}{\partial n} \to 0, \quad (7)
\]
where \(n\) is the symbol of coordinate that is normal to corresponding boundary of domain. Other boundary conditions depend on problem.

The function \(\bar{u} = f(\bar{y})\) determines an initial velocity profile. The function \(\bar{\phi} = \phi(\bar{y})\) is determined by known concentration of a polymer solution in ejector’s cross-section and other geometric characteristics of the ejector, namely, height of a disposition of its lower edge respectively streamlined surface \(\bar{h}\) and dimensionless width of a slot \(\bar{s}\). In case of homogeneous distribution of polymer concentration in ejector the last function may be presented by the following way
\[
\bar{\phi} = \begin{cases} 0 & \text{if } 0 \leq \bar{y} < \bar{h}; \\ 1 & \text{if } \bar{h} \leq \bar{y} < \bar{h} + \bar{s}; \\ 0 & \text{if } \bar{y} \geq \bar{h} + \bar{s}. \end{cases} \quad (8)
\]

CALCULATIONAL METHOD TECHNIQUE. The mentioned equations were transformed to the finite-difference form using a non-uniform rectangular grid. Usually, the nodes number of grid in direction of flow development was taken to \(i_{max} = 70-100\) and in normal to a surface direction \(j_{max} = 90-140\).

To unify the structure of calculating method the equations (2-4) have been presented into following generalized form
\[
A_1 \frac{\partial \phi}{\partial x} + A_2 \frac{\partial \phi}{\partial y} + S_0 + A_3 \left( F \frac{\partial \phi}{\partial x} \right) + A_4 \left( F \frac{\partial \phi}{\partial y} \right) = 0; \quad (8)
\]

The calculation of flows characteristics in the cross-section of \(x^{i+1}\) was done in two steps. The structure of equation (8) allowed transforming it to the three-point form.
\[ W\bar{\varphi}_{j-1}^{i+1} + P\bar{\varphi}_j^{i+1} + E\bar{\varphi}_{j+1}^{i+1} = R, \]  

where the indexes \( i, j \) determine the location of finite-difference grid nodes. Equation (9) links values \( \varphi^{i+1} \) in three adjacent nodes of a difference grid along \( y \). The factors \( W, P, E, R \) are functions of grid parameters and grid variables. The sequential application of equation (9) for all grid nodes in a calculated cross-section \( x^{i+1} = \text{const} \) reduces (9) to a system of the \( 2 \max_j - 2 \) linear algebraic equations with a three-diagonal matrix, which is solved by Thomas algorithm.

To solve the parabolic approximation of system (1-4) together with boundary conditions (5-7) the effective marching method with the second order accuracy has been elaborated. The general advantage of proposed finite-difference model equations and computational method is without-iteration two-step procedure, which allows quickly running along flow direction. In case of modeling problems connected with investigations of influence between near-wall flow and wall roughness elements or other obstacles the modifications of SIMPLE algorithms constructed on base of third order Leonard’s and TVD Zijlema’s schemes were applied. The numerical experiment have shown the workability of proposed generalizations of semi-empirical models of turbulence and effectiveness of elaborated methods of near-wall shear flows for the investigated here cases of flows.

The example of predictions of distributions for velocity and kinetic energy of turbulence in different sections of near-wall turbulent shear flow is presented by Fig. 1. This flow is characterized by presence of a small element like LEBU inside boundary layer that modifies both structure of mean flow and turbulent motion. Points on the Fig. 1 show the results of experimental measurements of corresponding profiles that have been obtained by Tulapurkara, Ramji and Radjacekar for this type of shear flow.

TURBULENCE MODELLING. With the aim of modeling the turbulent characteristics of investigated flows both algebraic and different variants of the \( k-\varepsilon \) differential models of turbulence have been used. These turbulence models have been modified under mentioned above circumstances. The modifications of this model by using functions of shift of logarithmic zone of velocity profile, which were proposed by author, allow the possibility to account the influence of roughness of streamlined surface and injecting of polymer solution into flow.

4. SOME PRACTICAL REALIZATIONS OF THE RECEPTIBILITY METHOD

On the basis of a method of receptivity the covering is developed, which efficiency was checked up at towing tests of model of a three-case vessel. On the external party of a covering the ledges were installed by lateral lines and located each other in the chess order. They are inclined under a positive angle of attack to a streamline flow. At movement of a vessel the micro-cavities are formed behind ledges. Formed bubbles of air move in area of local expansion of a flow of water in the channel between lateral surfaces of ledges of a covering and at a flow of ledges of the following line under action of increase of pressure are compressed. As a result - under the bottom side of a gliding vessel the gasous-water mixture is formed. The model trimaran had the common length equals to 1,9 m. The tests of model were carried out in development pool of the Institute of the hydromechanics NAS of Ukraine in a wide range of a towage speeds. Length of pool makes 50m, width - 6,8m, and the level of water during realization of tests made 2,5m. As criterion the Froude number, calculated on volumetric displacement, was used

\[ F_{Fr} = \frac{\varrho}{\sqrt{g\vartheta}} \]

For gliding courts at Froude number \( F_{Fr} < 1 \) the displacement mode of navigation is realized, at \( 1 < F_{Fr} < 3 \) - the transitional mode of movement is developed, and at \( F_{Fr} > 3 \) - the gliding mode is occurred. As researched size the amount of inverse hydrodynamic quality of model was used

\[ \varepsilon = \frac{R}{\Delta} \]

On Fig.3 the dependences of inverse hydrodynamic quality of the model of a gliding three-case vessel from the Froude number with the established covering and without it are submitted. It is visible, that for the displacement mode of navigation and significant part of a transitional mode ( \( F_{Fr} < 2,5 \) ) the coverings do not render appreciable influence on drag of water to movement of a vessel. In this case the forces of a wave nature and shape drag are dominant in common balance of hydrodynamic forces. At the Froude numbers \( F_{Fr} > 2,5 \) the role of friction drag dramatically increases in common balance of hydrodynamic forces acting on a vessel.
Therefore, the efficiency of a covering grows and in a mode of developed gliding the drag reduction (up to 14 %) of water to movement of model of a vessel is observed. At the Froude numbers $Fr_v > 4$ the efficiency of a covering are reduced due to increasing of spray resistance in intercase space. Other cases of flows behind the ledge were also realized.

**CONCLUSION**

In the present work in frames of model experiment the laws of disturbances development arising at cavity formation [1] are investigated, the various problems of turbulent flow over ledge are considered On the base of the receptivity method the various variants of practical realization of scientific researches are received. The obtained results can be applied as the tool of optimization of a construction and justified choice of regimes parameters of an equipment developed with the aim to provide the shear flows control.

**REFERENCES**