UNSTEADY CAVITATION IN THE IMPULSE AND WAVE PROCESSES

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ABSTRACT
In the paper, problems of the liquid pulse and wave movements calculation accompanying by the unsteady cavitation are discussed. Some models of the cavitation based on representations about a continuous medium are considered. Two models that are based on a constancy of i) pressure and ii) the sound speed in the cavitation zone are analyzed in details.

The numerical calculations of the cavitation require or allocation of the cavitation boundaries (method of the characteristics), or addition of new types of "breaks disintegrations" (Godunov’s method) what complicates the algorithm. A new numerical scheme that is free from the listed lacks is offered. It is based on introduction of an artificial viscosity in the movement equations of an ideal compressed liquid. The results of calculations for different cavitation models are compared with analytical solutions.

The calculation results for various hydro-pulse installations are given: generator of pulse jets, percussive hydro-cannon, installation of compression, press-gun, and installation for determination of the water dynamic stretching durability.

INTRODUCTION
The occurrence of the stretching stresses in a liquid at the certain instants is a characteristic feature of the pulse and wave liquid movements. In a result of them, the unsteady cavitation arises. The occurrence of the cavitation is connected to the rarefaction waves which arise, for example, by reflection of the compression waves from a free surface, or by breaking the piston in the hydro-cannon, installation of compression, and press-gun [1 - 3]. If one calculates in a usual way, stretching stresses can reach tens Megapascals, they acting for less than a millisecond. As it is known, the usual water practically does not bear the stretching stress and it collapses. It means that the unsteady cavitation arises. The magnitude of a critical stretching stress by which water collapses makes up about 0.05 - 1 MPa [4 - 6]. The stretching stress up to 28 MPa was reached [6] for a specially processed water by a stationary stretching.

NOMENCLATURE

\[ e \quad \text{specific energy} \]
\[ m \quad \text{mass} \]
\[ F \quad \text{nozzle cross-sectional area} \]
\[ p \quad \text{water pressure} \]
\[ R \quad \text{nozzle radius} \]
\[ q_1, q_2 \quad \text{artificial viscosity} \]
\[ t \quad \text{time} \]
\[ u, v \quad \text{speed} \]
\[ x, r \quad \text{coordinates} \]
\[ \lambda_1, \lambda_2 \quad \text{artificial viscosity coefficients} \]
\[ \rho \quad \text{density} \]

Subscripts
\[ 0 \quad \text{initial conditions} \]
\[ f \quad \text{free surface} \]
\[ c \quad \text{cavitation} \]
\[ cr \quad \text{critical parameters} \]
\[ p \quad \text{piston} \]
\[ s \quad \text{nozzle exit} \]

1. THE UNSTEADY CAVITATION MODELS
The most widespread state equation of water is an experimental one in the Teit’s form

\[ p = B\left(\frac{p}{\rho_0}\right)^n - 1. \] (1.1)

Here \( B = 304.5 \text{ MPa}, n = 7.15, \rho_0 = 1000 \text{ kg/m}^3 \) are empirical constants. It actually represents the isentrope equation and gives good results to pressure up to 30 GPa. If pressure is more, they use the binomial and trinomial state equations, correspondingly

\[ p = nB\left[\frac{p}{\rho_0}\right]^n - 1 + \rho(n - 1)e, \] (1.2)

\[ p = a(p/p_0 - 1) + b(p/p_0 - 1)^2 + cpe, \] (1.3)

where \( e \) is the internal energy of a mass unit; \( a, b, c \) are empirical constants [7].

The given state equations do not take into account an opportunity of the cavitation occurrence. These equations formally suppose existence of large negative pressures (stretching) in water. But water does not keep such a pressure,
and its structure is broken. If the process is unsteady, the unsteady cavitation arises. Its reason is the rarefaction waves. Neglect the cavitation results in essential quantitative and qualitative distortions of the process while calculating.

There are some models of the cavitation [3]. Let us consider two models when pressure and the sound speed are constant in the cavitation zone. In the first one, it is considered that the water pressure is equal to a critical one \( p_{cr} \). It may be both \( p_{cr} > 0 \) and \( p_{cr} < 0 \), that is, water maintains small stretching stresses (negative pressure). It is considered that the cavitation arises and exists if the water density is less than a critical one \( \rho_{cr} \). The water state equation (1.1) takes the form

\[
 p = \begin{cases} 
 B(p/p_0)^\gamma - 1, & \rho > \rho_{cr}, \\
 0, & \rho \leq \rho_{cr}.
\end{cases} \tag{1.4}
\]

Medium particles fly away and move independently from each other in the cavitation zone. The sound speed is equal to zero. As soon as density becomes equal to \( \rho_{cr} \), the water structure rehabilitates itself by jump, and the cavitation zone is closed.

The state equation (1.3) takes into account the cavitation as follows

\[
 p = \begin{cases} 
 a(p/p_0 - 1) + b(p/p_0 - 1)^2 + \text{cpe}, & \rho \geq 0; \\
 \rho_{cr}, & \rho < 0; \end{cases}
\]

\[
 \rho_{cr}, \rho \leq \rho_{cr}. \tag{1.5}
\]

In the second model, it is considered that the sound speed is constant in the cavitation zone. The state equation has the form

\[
 p = \begin{cases} 
 p_0 + a^2(p - p_0), & \text{если } \rho > \rho_{cr}; \\
 \rho_{cr} + a_0^2(p - \rho_{cr}), & \text{если } \rho \leq \rho_{cr}, \end{cases} \tag{1.6}
\]

where \( p_0 = 0 \) and \( \rho_0 \) are the initial water pressure and density.

### 2. Calculation of the Unsteady Cavitation

To calculate the unsteady cavitation the method of the characteristics and the Godunov’s method was used [6, 7]. Its calculation with a method of the characteristics demands allocation of the cavitation zone boundaries what makes the algorithm not uniform. If one calculates with the Godunov’s method, additional new types of "breaks disintegrations" arise that complicates the algorithm.

To describe the flow uniformly without an allocation of the breaks, an artificial viscosity \( q \) (viscous pressure) is entered in the equations of movement as an addition to the hydrostatic pressure. For a one-dimensional flow, the movement equation system in the Lagrange form is the following [8]

\[
 \frac{dp}{dt} = -\rho \frac{\partial u}{\partial x}, \quad \frac{du}{dt} = \frac{1}{\rho} \left( \frac{p + q}{\partial x} \right), \quad \frac{d\epsilon}{dt} = \frac{(p + q) \partial u}{\partial x}. \tag{2.1}
\]

Viscous pressure is presented with a sum of a linear term \( q_1 \) and a square-law one \( q_2 \) [9]

\[
 q_1 = \lambda_1 \rho \Delta x \left( \frac{\partial u}{\partial x} \right)^2, \quad q_2 = \lambda_2 \rho \Delta x \left( \frac{\partial u}{\partial x} \right)^2.
\]

Here \( \lambda_1 \) and \( \lambda_2 \) are experimental factors, \( a \) is the sound speed, \( \Delta x \) is the grid step. The square-law viscosity is included on shock waves, and the linear viscosity does only to smooth pulsations. The optimum values of the viscosity factors were established experimentally: \( \lambda_1 = 0.6 \) and \( \lambda_2 = 2 \).

The approximation of the equations (6) is made on a Lagrange grid under the formulas

\[
 u_{i+1}^{(1)} = u_i - \frac{2 \Delta x}{\rho_0} \left( \frac{p + q}{\partial x} \right)_{i,1/2} + \left( p + q \right)_{i,1/2},
\]

\[
 \rho_{i+1/2}^{(1)} = \rho_{i+1/2} \frac{\Delta x_{i+1/2}}{\Delta x_{i,1/2}},
\]

\[
 \epsilon^{(1)}_{i+1/2} = \epsilon_{i+1/2} - \Delta t \frac{p + q}{\rho_0} \left( \frac{u_{i+1} - u_i}{\Delta x_{i,1/2}} \right)^2,
\]

\[
 x_{i+1} = x_i + u_{i+1/2} \Delta t.
\]

The speed is determined at the grid knots with coordinates \( x_i \), and other parameters are determined at points with coordinates \( x_{i+1/2} = (x_i + x_{i+1})/2 \).

The offered method (VIS) was tested while calculating a number of modeling tasks some of which have the analytical decision. The balances of mass, pulse and energy were carried out with accuracy of 0.1 \%. The comparison of the VIS method with the method of the characteristic and the Godunov’s ones has shown the complete coincidence of the results for different configurations of the shock, compressed and rarefaction waves.

Let's consider a problem, in which a motionless liquid \( p_1 \), at the initial moment, the diaphragm is broken off, and the liquid comes in a diaphragm on the right, is under pressure \( p_0 \). At the initial moment, the diaphragm is broken off, and the liquid comes in movement. The initial and boundary conditions are the following

\[
 u(0, x) = 0, \quad p(0, x) = p_1, \quad p(0, x) = p_1, \quad 0 < x < L, \tag{2.3}
\]

\[
 u(t, 0) = 0, \quad p(t, x_f) = p_0. \tag{2.4}
\]

After break off the diaphragm, a rarefaction wave is distributed toward the wall and then it is reflected from the wall as a rarefaction wave too. As a result, the unsteady cavitation arises near the wall. If \( n = 3 \) in equation (1.1), it is possible to obtain an analytical solution [8]. The liquid speed and density in the cavitation zone change under the laws

\[
 u = \frac{x}{t}, \quad p = \rho_0 \frac{L}{a^2 t}, \quad 0 \leq x \leq u_a t. \tag{2.5}
\]
Here $u_0$ is the liquid speed after rarefaction, coordinate $x$ is counted from the wall. The speed in the cavitation zone increases linearly from zero at the wall up to $u_0$ at the cavitation boundary, which moves with the same speed.

In Fig. 1, the distributions of the speed and density, calculated with the help of the VIS method at instants $t = 1, 2, 3, \text{ and } 4$ (curves 1-4), are given according to the first cavitation model with constant pressure. The circlets show the analytical solution. The critical pressure and density values were equal to zero. All the magnitudes are dimensionless according to the scales: time $t_c$ when the cavitation arises, speed $u_0$, length $L$, density $\rho_0$.

Fig. 1. Distribution of a) speed and b) density

To investigate the cavitation closing, the same problem was used. Stop of the liquid free surface was modeled. This means that a steady shock wave moving toward the cavitation zone arises. On reaching the cavitation zone, the wave begins to close it and becomes unsteady as the parameters in the cavitation zone depend on space and time.

In Fig. 2, the distributions of the pressure, speed and density (curves 1, 2, and 3) at instant $t = 2.5t_c$ are given. All the parameters suffer their breaks at the cavitation boundary. It means that the cavitation zone is closed by a shock wave. The shock wave speed in the cavitation zone is much less than the one in the liquid; the average speed of the wave is equal to about $0.1c_1$.

The second cavitation model with a constant sound speed was also investigated. In this case, the pressure and density in the cavitation zone are connected with equation (1.6). The critical value of the sound speed is chosen by means of analysis of experimental and theoretical researches of the acoustic waves distribution in water containing babbles. Even if the babbles concentration is small, the sound speed in such a water is smaller than the one in the air. The minimal value of the sound speed equal to 30 m/s was taken as the critical one $a_{cr}$.

In Fig. 3, the speed and density distributions at instant $t = 2t_c$ for different $a_{cr}$ are given: curve 1 corresponds to $a_{cr} = 30$ m/s, curve 2 does to $a_{cr} = 6$ m/s, curves 3 does to $a_{cr} = 120$ m/s. the continuous curves show speed, the dotted ones do density, the circlets do the solution according to the first cavitation model. As one can see, the calculations results according to different models completely coincide for the least critical sound speed (curves 2 and circlets on it). Distinction in distribution increases (curves 3) with the sound speed increase as the second model assumes an interaction of the medium particles in the cavitation zone and, consequently, wave processes.

It is visible that the numerical solution completely coincides with analytical one for the cavitation development what confirms the reliability of the VIS method.

3. NUMERICAL CALCULATION

3.1. Percussive hydro-cannon

The unsteady cavitation arises in the percussion hydro-cannon [1, 10]. The numerical calculations of the cavitation require or allocation of the cavitation boundaries (method of the characteristics), or addition of new types of "breaks disintegrations" (Godunov’s method) what complicates the algorithm. The VIS method described in the second section is free from these lacks [11, 12].

To calculate with the Godunov’s method the following quasi one-dimensional equations with the initial and boundary conditions are used
\[
\frac{\partial pF}{\partial t} + \frac{\partial \rho u F}{\partial x} = 0, \\
\frac{\partial u}{\partial t} + \frac{\partial}{\partial x} \left( u^2 + \frac{n}{2} \frac{p + B}{\rho} \right) = 0, \\
u(0, x) = 0, \quad p(0, x) = 0, \quad x \in [-L, 0], \\
u(t, x_p) = u_p, \quad p(t, x_f) = 0,
\]
where \( F \) is the nozzle cross-sectional area; \( x_p \) and \( x_f \) are the piston and free surface coordinates; \( u_p \) is the piston speed; \( L \) is the water charge length. The movement of the piston is described with the following equations with the initial conditions
\[
\frac{du_p}{dt} = -\frac{F_p}{m_p} p; \quad \frac{dx_p}{dt} = u_p, \\
u_p(0) = u_0; \quad x_p(0) = -L.
\]
Here \( F_p \) is the piston cross-sectional area, \( p_p \) is pressure on the piston, \( m_p \) is the piston mass, \( u_0 \) is the piston initial speed.

To calculate with the artificial viscosity method the following equations are used
\[
\rho \frac{d\rho}{dt} + \rho \frac{d\rho u}{dt} + \rho \frac{d\rho F}{dt} = 0, \\
\rho \frac{du}{dt} + \rho \frac{d\rho u}{dt} = 0, \\
\rho \frac{d\rho}{dt} + \rho \frac{d\rho F}{dt} = 0.
\]

3.2. Installation of compression

A press-gun, in which the kinetic energy of the piston is used, is described in work [3]. Such a gun is a device for impulse punching. The processes of the metal hydro-pulse punching, as a rule, are accompanied by the unsteady cavitation, which can arise both on the half-finished product and piston, and are described with the same equations (3.1) or (3.4) depending on the method being used as for the hydro-cannon. The initial and boundary conditions are the following
\[
u(0, x) = 0, \quad p(0, x) = 0, \quad x \in [-L, 0], \\
u(t, x_p) = u_p, \quad u(t, 0) = 0.
\]
The piston movement is described with equations (3.3).

There are some results of calculations below. They are given for the installation with the following data [3]: length of the water column \( L = 650 \text{ mm} \), the piston radius of 56 mm, the piston mass of 2,3 kg, the piston initial speed of 32 m/s.

In Fig. 5, results of calculations, carried out with the Godunov’s method with the cavitation (continuous lines) and without it (dotted lines), are shown: 1 - pressure on the piston, 2 - pressure on the half-finished product, 3 - the piston speed.

In Fig. 6, comparison of calculations executed for the cavitation with different methods is given. The continuous lines correspond to the Godunov’s method, circle dots to the artificial viscosity method. The curve numbering is the same as it is in Fig. 5. As it is visible, the calculations results coincide at all.

3.3. Installation for research of the water dynamic durability to break

The installation to research the water durability to break by dynamic loadings [2] is a cylinder filled a liquid. The cylinder is closed from an end face, and a piston is inserted into it from another end. The piston is connected to other piston of a greater diameter, which can move with a big acceleration under action of the compressed gas whose pressure is supported constant by a...
receiver. When the pistons are actuated, a rarefaction wave is distributed in water, pressure in which goes down linearly in time. A sensor registers the water pressure at the end face. The indications of the sensor change by jump when the cavitation arises. Changing pressure upon the piston, it is possible to adjust the pressure change speed and to investigate dependence of the cavitation occurrence on the stretching stress action time. The maximal acceleration of the piston reached 170 g. Usual water without bubbles which are visible with the naked eye was investigated. It is experimentally established that the critical pressure depends on time: the less the action time is, the more the critical pressure is. For example, the cavitation arises by the critical pressure equal to -0.25 MPa if the action time is equal to 50 ms.

Fig. 7 shows dependences of pressure upon the piston and the closed end face, as well as the piston speeds on time, which are calculated with taking into account the cavitation according to the first model, \( p_c = -0.25 \) MPa (curves 1, 2, and 3). As one can see, the pressure at the piston decreases linearly from zero to its critical value for approximately 50 µs as it was registered experimentally.

### 3.4. Pulse jets generator

The installation being researched [13] represents a cylinder chamber of diameter \( D \) and length \( L \) filled with water. At one end, a piston is located, at another its end there is an aperture of a small diameter \( d \) from which a pulse water jet flows out. At the initial instant, the piston is actuated by impact; the water is compressed and begins to flow out as a pulse jet. The movement of water is described with the equations system (3.1) with the following initial and boundary conditions:

\[
\begin{align*}
    u(0, x) &= 0, & \rho(0, x) &= 0, & 0 \leq x \leq L; \\
    u(t, x_0) &= u_p, & u(t, L) &= 0.
\end{align*}
\]

The movement of the piston is described with equation (3.3). The outlet speed is determined after the Bernoulli equation.

Some results of calculations are given below. Fig. 8 shows the diagrams of the time dependence of pressure at the piston and at other end of the installation, the piston speed and outlet speed calculated with the artificial viscosity method (curves 1, 2, 3, and 4 accordingly). The data of the installation are taken from work [12]: \( D = 25 \) mm, \( d = 2.5 \) mm, \( L = 75 \) mm, \( u_p = 3.37 \) m/s, the water mass \( m = 0.088 \) kg.

The pressures at the piston and at other chamber end change spasmodically. The piston is quickly broken, letting out the rarefaction waves. The wave is reflected from other end and returned to the piston. To the returning the wave to the piston, the pressure at it decreases almost twice, and then falls down more intensively than before the arrival of the reflected wave (curve 1). It is visible from the diagrams, the cavitation arises at both ends of water to instant \( t = 250 \) µs. The fractures on the piston speed diagram correspond to the arrival of the next compression wave to the piston.

### 3.5. Dynamics of a spherical shell

The movement of a thin spherical shell filled with water caused by the electric discharge in the center of the shell was investigated. The motion of the water and shell is described with the equations:

\[
\begin{align*}
    \frac{\partial p}{\partial t} + u \frac{\partial p}{\partial r} + \rho \frac{\partial u}{\partial r} &= -\frac{2\mu}{\rho} \frac{\partial u}{\partial r}, \\
    \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial r} &= \frac{1}{\rho} \frac{\partial p}{\partial r}.
\end{align*}
\]
\[
\frac{d^2 w}{dt^2} + \frac{2E}{\rho_c R_c^2 (1 - \mu)} w = \frac{p_c}{\rho_c \delta_c}.
\]

Here \( w \) is the shell transition; \( E \) is the Jung’s module; \( E \) is the Poisson’s factor; \( R_{c0} \), \( R_c = R_{c0} + \omega \), \( \delta_c \) and \( \rho_c \) are the initial and current shell radius, shell thickness, its material density; \( p_c \) is the water pressure upon the shell. The material was rubber.

In Fig. 9, the dependences of the pressure and the shell speed on time (curves 1 and 2) are given. The shell becomes move after the arrival of the pulse of pressure to it, and the cavitation arises near it face. The cavitation disappears in due time.

\[ p, \text{ MPa} \]
\[ u, \text{ m/s} \]

\[ 0 \quad 250 \quad 500 \quad 750 \]
\[ t, \text{ mks} \]

Fig. 9. Dependence of parameters on time

3.6. The liquid jet outflow if its expansion is not full

The unsteady cavitation can also arise in steady processes if they have a wave nature. It is possible for supersonic liquid flows which are described with the equations of the hyperbolic type.

The outflow of a supersonic liquid jet is accompanied by the cavitation [14, 15] if its expansion is not full. Let us consider the following problem. A liquid jet with speed \( u_0 \) flows out a cylindrical nozzle with radius \( R_s \) in vacuum. At the initial instant, a flat steady shock wave with pressure \( p_0 \) behind it reaches the nozzle outlet and leaves for the jet. Parameters of the shock wave are such that the outflow remains supersonic. The liquid is considered to be ideal and compressed, and the flow is considered to be isentropic. The beginning of the coordinate is disposed at the nozzle outlet; an axis \( x \) is directed along the jet.

In the accepted statement, the axially symmetric liquid movement is described with the equations system of unsteady gas dynamics with the following initial and boundary conditions

\[ \frac{\partial p}{\partial t} + \frac{\partial p u r}{\partial x} + \frac{\partial p v r}{\partial r} = 0, \]
\[ \frac{\partial p u r}{\partial t} + \frac{\partial p u^2 + p v^2}{\partial x} + \frac{\partial p v r}{\partial r} = 0, \]  

(3.6)

\[ \frac{\partial p v r}{\partial t} + \frac{\partial p v u r}{\partial x} + \frac{\partial p v^2}{\partial r} = p, \]

\[ u = u_0, \quad v = 0, \quad p = 0, \quad 0 < x, \quad 0 \leq r \leq R_s, \]

\[ u = u_1, \quad v = 0, \quad p = p_1, \quad 0 \leq r \leq R_s, \quad \rho_{0G} = 0, \]

(3.7)

(3.8)

where \( u_1 \) is speed behind the shock wave; \( G \) is the jet free surface.

The second cavitation mode was accepted.

The problem was solved numerically by the Godunov’s method as an unsteady one till then the process becomes steady (the being settled method). Some results of calculations for \( p_1 = 0,5, \quad u_0 = 1,256 \) what corresponds to the outlet Mach number \( M_a = 1,11 \) and the jet dynamic head \( p_0 = 8,2 \), are submitted below. All the magnitudes are dimensionless according to scales: nozzle radius \( R_s \), sound speed \( a_0 \), time \( R_s / a_0 \), and pressure \( p = B \).

In Fig. 10, the isobars (continuous curves, the pressure values are given near the curves) are shown at the instant when parameters have been settled.

The numerical solution (dotted curves) for a flat problem is compared to the analytical solution (a stroke dotted curves) received in [14]. The free surface and the cavitation zone boundaries (curves 1 and 2) for the numerical solution are given at instant \( t = 3 \) and illustrate the establishment of the flow. The cavitation is developed in the jet center. The cavitation zone occupies approximately a third part of the jet cross section.

In Fig. 11, the speed distribution along the jet axis at the different instants is given. Curves 1 and 2 correspond to instants \( t = 0,5 \) and 2, and curve 3 corresponds to the established distribution. Pressure upon the jet axes reduces to zero at \( t = 1 \).

Fig. 10. To the calculation of the jet outflow

Fig. 11. Distribution of speed along the jet
The liquid flows from the jet center, and the cavitation develops here. The liquid speed increases at the jet axes reaching the limiting value that corresponds to its full expansion (curves 2 and 3). The jet full expansion occurs on distance $x = 0.6$ from the nozzle outlet.

CONCLUSIONS

Some models of the unsteady cavitation are considered and analyzed. It is shown, in what way the cavitation is taken into account while calculating. A method of the artificial viscosity is offered to calculate flows with the cavitation. The calculation results show that ignoring the unsteady cavitation can essentially distort the flow picture.

The methods developed allow calculating different pulse and wave processes accompanied by the unsteady cavitation in one- and two-dimensional approach.

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