SUPERCavitATION FLOWS WITH GAS INJECTION  
- PREDICTION AND DRAG REDUCTION PROBLEMS

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ABSTRACT
The investigation results of the supercavitation for high speed motion of prolate axisymmetric bodies in water are presented. The cavitation drag forming and drag reduction problems are considered on base studying of the peculiarities of the supercavitation flows for cavities with given pressure and with gas injections. Investigations are developed on the ground of the simple heuristic models, integral conservation laws, Slender Body Theory and another ordinary here approximations.

Keywords: supercavitation, cavity, drag, drag reduction

INTRODUCTION
Application of the supercavitation give the possibility, isolated body against water, to avoid considerable viscous drag. In doing so the least drag coefficients for the cavity middle section $C_D$ (the body close enough inscribed in the cavity) here are reached in case of considerably prolate cavities. Possibilities of $C_D$ decreasing are limited by maximal bodies aspect ratio which can provide its strength. Values of $C_D \sim 0.05-0.001$ and lees here are real and for some cases drag in water can be comparable even with drag in case of the motion in air.

With account of importance of this potentates key attention of the consideration is directed to the cavitation drag forming and drag reduction problems, close connected here too with the supercavitation flows forming and prediction problems. The possibilities of the cavity form changing under internal pressure changing and gas flow in the cavity influence are studied. The attempt to develop the general characteristics of the supercavitation motion effectiveness are undertaken.

Considerable part of the theory is connected with the models of ideal incompressible fluid.

1. SUPERCavitation FOR GIVEN PRESSURE

1.1 To the statement of the problem
The solution of the problem in case of cavitation flow here in the general case is defined on base complicated enough nonlinear problem with unknown free boundaries. The most typical is statement for the unlimited steady undisturbed at infinity flow of ideal incompressible fluid with free boundaries and using of the Raibysinsky closer scheme. Flow potential is described by Laplace equation. For known cavitator surface impenetrability condition is given. For unknown cavity surfaces impenetrability and desired pressure $P_c$ conditions are given. Flow perturbations are damped at the infinity. Investigations here are reflected by many of publications and in particular [1-3] and others.

1.2 Slender axisymmetric cavity diffe-Rential equation
Thank to $C_D$ values are small namely for prolate cavities the case of the slender cavities is especially important and at the same time slenderness property is as strong simplification factor too. This fact gave the possibility starting from [4] on base known Asymptotic Expansion Method (MAEM) and Slender Body Theory (SBT) [5] to create known linear theory of the flat cavitation flows and after that linearized theory of the axisymmetric flow too [6-8] and others. Within accuracy of till small values $\delta^2\ln1/\delta^2$ the general statement of the problem can be transformed into the integer-differential equation (IDE) for slender axisymmetric cavity (here $\delta$ is slenderness parameter for cavitator and cavity as whole as some slender body). In particular in case of ordinary steady cavity $r = R(x)$ behind small alike disc cavitator this equation is [6-8]:
\[
\left( \frac{dR}{dx} \right)^2 + \frac{d^2R^2}{dx^2} \ln \frac{2x(L-x)}{R} - \frac{1}{4} \frac{d^2R^2}{dx^2} - \frac{d^2R^2}{dx^2} = \frac{\Delta P(x)}{\rho}.
\]

Here \( r, x \) - cylindrical coordinates system connected with moving cavitator, \( \Delta P(x) = (P_{m} - P_{o}) \) - pressure difference between hydrostatic pressure in the flow and in the cavity, \( \sigma(x) = (P_{m} - P_{o}) \frac{\rho U^2}{2} \), \( U_{m} \) - speed at infinity, \( L \) - cavity length, \( \rho \) - fluid mass density. Under every equation the term orders for \( \delta \rightarrow 0 \) are indicated. Unsteady (IDE) [6-8] is alike.

(SBT) approximation realize simple physical model of the radial flow near expanding in the motionless fluid slender cavity (body) sections Fig. 1. General case 3-D unsteady flow here is approximated by flat flow of cylindrical expansion in the finite near slender body (cavity) surface zone \( r = \Psi(x,t) \). For particular case for ordinary steady cavity of constant pressure behind small like as disc cavitator this value in the middle section is \( \Psi = -0.6 \pm 0.7 \lambda_{k} \) (\( \lambda_{k} \) - cavity semi-length). The process of the slender axisymmetric cavity \( r = R(x,t) \) creation (here \( r, x, t \) - cylindrical coordinates, time) can be illustrated by simple physical model Fig.1. For slender cavities alike as disc cavitator is small as compared to cavity sizes and cavitator drag is practically independent on cavity form. Moving with speed \( U_{n} \) in motionless fluid cavitator push fluid to aside and cavitator drag is transformed into the kinetic energy of radial flow in every passing through by it fluid section. Further expanding of created radial flow by inertia take place under action of the hydrostatic and pressure in the cavity difference \( \Delta P(x,t) = (P_{m} - P_{o}) \) which in general case can be as alternate. At the middle section all kinetic energy is transformed into the potential and further closing of the cavity process is prolonged under action of the pressure difference. This model is really strip model and in the coordinate system connected with motionless fluid is presented by the equation [9, 10]:

\[
\mu \frac{d^2R^2}{dx^2} + \frac{\Delta P(x)}{\rho} = 0,
\]

And

\[
R^2 |_{x=0} = 0, \quad \frac{dR^2}{dx} |_{x=0} = \frac{2 \pi \mu D}{\sqrt{k \pi \mu U^2_{m}}}.
\]

Here the coordinate system is connected with moving cavitator. \( D \) - cavitator drag calculated by quasi stationary way, \( k \approx 0.95 \pm 1 \) small correction accounting prolong energy transportation. Equations (3, 4) express limit properties of the section independent expansion and absent of prolong energy transportation along flow near slender cavity. There are alike unsteady equations system too [9-11].

### 1.3 Steady cavity form and sizes

For \( P_{m}, P_{o}, \sigma = \text{const} \) equation (3, 4) define known ellipsoidal cavity in the form:

\[
R^2 = R_{n} \left( \frac{2c_{d}}{k\mu} \right) x - \frac{\sigma}{2\mu} x^2.
\]

Here \( R_{n}, c_{d} \) - cavitator radius, drag coefficient, in particular for disk \( c_{d} = c_{d0}(1 + \sigma), \ c_{d0} = 0.82 \). Solution (5) define known
formula for cavity maximal radius $R_k$ and characteristic dependencies for cavity semi-length $L_k$ and aspect ratio $\lambda_k$:

$$R_k = R_n \frac{\sqrt{cd}}{\sqrt{k}\sigma} , \quad L_k = R_n \frac{\sqrt{cd} 2\mu / k}{\sigma} , \quad \lambda_k^2 = \frac{2\mu}{\sigma}. \quad (6)$$

Base idea of the type (3-4) equations creation is to use limit form and properties of this equations but to improve considerably its accuracy with help of appropriate $\mu, k$ values on base more accurate equations of type (1). Values $\mu, k$ in equations (3, 4) have to the limit properties to depend on aspect ratio only but really this properties are realized as very slowly changing functions on the aspect ratio and cavity form. This fact give the possibility to define $\mu, k$ on base more accurate solutions for some base case - to apply after that this values for calculation in wide enough range which would be some near to this case. One of the most typical is ordinary steady cavity for $\sigma = \text{const}$ in case of alike disc cavitator. Second order asymptotic solution of Eq. (2) for $\delta \rightarrow 0$ within account of dependence (6) for $\lambda_k$ define $\mu$ value as [6, 7]:

$$\lambda^2 = \frac{2}{\sigma} \ln \frac{\lambda}{\sqrt{e}} , \quad \mu = \frac{1}{2} \ln \frac{\lambda}{\sqrt{e}}. \quad (7)$$

More perfect dependence which can be applicable till up $\lambda \sim 3 - 4$ are (here dependence (8) for $\lambda$ is on the base known P. Garabedian formula) is:

$$\lambda^2 = \frac{1}{\sigma} \ln \frac{1.5}{\sigma} , \quad \mu = 1 \ln \frac{1.5}{\sigma}. \quad (8)$$

$$\lambda^2 = \frac{2}{\sigma} \ln \left( \frac{\lambda}{\sqrt{e}} \sqrt{1 + \frac{1}{\ln \lambda^2}} \right) , \quad \mu = \ln \left( \frac{\lambda}{\sqrt{e}} \sqrt{1 + \frac{1}{\ln \lambda^2}} \right). \quad (9)$$

Fig. 2, 3 present calculation results for values $\lambda, \sqrt{\mu}$ on base dependencies (7-9) in comparison with nonlinear numerical calculation cavity behind disc for Riabyshinsky closer [12].

1.4 Cavity energetic characteristics

Kinetic energy of fluid $E_k$ in the section of the flow near cavity in particular in the steady case is defined by expression:

$$E_k = \frac{\mu \rho U_0^2}{4} \left( \frac{dR}{dx} \right)^2. \quad (10)$$

In doing so the equation (3) can be presented as energy conservation equation in the fluid section:

$$k\pi \frac{\mu \rho U_0^2}{4} \frac{dR}{dx} \left( \frac{dR}{dx} \right)^2 + \pi \frac{dR}{dx} \Delta P(x) = 0, \quad (11)$$

where energy is given by cavitator to the fluid section for the initial moment is defined by expression:

$$E_k = \frac{\mu \rho U_0^2}{4} \left| \left( \frac{dR}{dx} \right)^2 \right|_{x=0} = \frac{D}{k}. \quad (12)$$

The most evident is case of $\sigma = \text{const}$ where equation (3) express conservation of whole kinetic energy of radial flow $E_k$ and

$$kE_k + E_p = k\pi \frac{\mu \rho U_0^2}{4} \left( \frac{dR}{dx} \right)^2 + \pi R^2 \Delta P = D. \quad (13)$$

1.5 Cavities of alternative pressure

Vertical cavity here is typical. In this case
$\Delta P = (P_\infty - P_c) = (P_0 - P_c) \pm \rho g x$ (g - gravity, $P_0$ - hydrostatic pressure in the cavitation section). The solution for steady cavity form on base (3, 4) is [13]:

$$R^2 = \frac{2c_0^2}{k\mu} \left[ -\sigma_o x^2 + \frac{1}{3} \frac{Fr^2}{\sigma_o} x^3 \right], \quad (14)$$

where: $R = R/R_n$, $\sigma_o = \frac{P_\infty - P_c}{\rho U_\infty^2 / 2}$, $Fr^2 = U_\infty^2 / gR_n$. Note here possibilities of $\sigma$ negative values. Essential can be influence gravity on $\mu$ value. Important is relation [13]:

$$\sigma_o Fr^2 = 4/3, \quad (15)$$

where - $Fr^2 = U_\infty^2 / gL$, $L$ - cavity length. Relation (15) is condition of the sharp edges appearing at the cavity nose zones or end. Ideally this conditions accordingly mean possibility of the cavity creation without of cavitation drag and of kinetic energy zero value at the cavity end which can transform into to the wake behind cavity.

Fig. 4 illustrate ordinary and vertical cavity form features. Note here known investigations by A. Acosta, C. Leno and R. Street, O. Kiselev and others.

1.6 Possibilities of given pressure cavities prediction

Important here is nonlinear numerical calculation methods for classical statement on base of the model of ideal incompressible fluid with Ribiashinsky closure for the disc. Hopeful enough solutions methods have been developed started from known investigations by C. Brennen, L. Guzevsky, A. Terentiev and others. Equations (3-4) describe cavity form rough enough, however very good predict it's main sizes and volume. Independent of the sections expansion with account of Eq. (2) take place for unsteady flows and there are here alike unsteady equation system of type (3-4) [9-11]. In this case for calculation in the most typical range of $\lambda \sim 8 \pm 16$ constant value of $\mu \sim 2$ usually is applied. In spite of essential solutions on base Eq. (3-4) roughness, these equations are of considerable importance thank to simplicity, universality and direct connection with energetic characteristics of the flow. But it is essential to note that this equations rough enough describe forward part of cavity. For steady flow oriented structure of the dependencies for volume gas losses $Q_{c \text{- out}}$ (17a) for pressure corresponding to the pressure in a cavity and mass gas loss from cavity $Q_{m \text{- out}}$ (17b) can be estimated in the form:

a) $Q_{c \text{- out}} = k_q c_d R_n^2 U_\infty \frac{1}{\sigma^2}$

b) $Q_{m \text{- out}} = k_q \bar{P}_\infty c_d R_n^2 U_\infty \frac{1 - \sigma/E}{\sigma^2} \rho_a$, $\bar{P}_\infty = P_0 / \rho_a \quad (17)$

Here $E = 2P_\infty / \rho U_\infty^2$ - Eiler Number, $P_a, \rho_a$ are pressure and gas mass density what correspond to nature pressure of 1atm. $k_q$ - const usually is defined on base experiments for ordinary steady cavity behind disc. More precise theory and gas loss dependencies have been defined in the [16]. The gas loss constant $k_q \sim 0.0147$ is estimated based on the universal dependence for gas loss [16] with account of the date [17]. For transformation of (17a) into (17b) isothenorm law is applied. Gas loss value can be strongly depended on action of horizontal and vertical gravity components. Important here is closer type which can be of natural or closure on definite body.

Key moment for defining of the unsteady cavities is to define pressure in the cavity $P_c = P_c(t)$ and after that cavity can be found on base equation (3, 4) or by any another way. In order to define $P_c = P_c(t)$ the most effective is application of unsteady equations of type (3, 4) together with simplest polytropic dependence for gas in the cavity and add equation of the mass in the cavity conservation with using quasi stationary process [10, 11]. There statement of this problem with account thermodynamic processes too[18]. The most typical for ordinary gas injection are two points:

- For not high enough speed of gas flow in the cavity influence of the flow on the cavity form is not essential, pressure of gas $P_c = P_c(t)$ is constant along whole volume.
Cavity contained gas is elastic oscillated system. Complete enough theory on base equations of type (3, 4) have been developed here by E. Paryshev [16]. Note here too investigations by E. Silberman and C. Song, J.M, Michel, J.P Franc and others.

Very important here gas loss problems connected with problems of cavity back part forming, thermodynamics processes, vortex creations and another effects [ 20-22]. For ordinary gas injection possibilities to change cavity form are very limited. Essential changing of cavity form are possible in the narrow enough gap between body and cavity only. Asymptotic solution for gas flow in the cavity was obtained on base model of ideal incompressible fluid for given pressure \( P(x) \) on the cavity surface. Second order solutions for gas flow potential \( \Phi \) for \( \delta \to 0 \) is [23]:

\[
\frac{d^2R^2}{dx^2} + \frac{\sigma_{cl}}{\mu} \left( \frac{1}{\rho_{cl} U_{cl}^2} \right) \left[ - \left( \frac{R_1^2 - B_1^2}{R^2 - B^2} \right) \right]^2 = 0 . \tag{19}
\]

Here \( \rho_{cl} \) - gas mass density in the cavity, \( P_{cl}, U_{cl} \) pressure and speed of gas at the some initial point. For steady flow equation with account of low speed gas in the cavity on base Eq. (3) is:

\[
\frac{d^2R^2}{dx^2} + \frac{\sigma_{cl}}{\mu} \left( \frac{1}{\rho_{cl} U_{cl}^2} \right) \left[ 1 - \left( \frac{R_1^2 - B_1^2}{R^2 - B^2} \right) \right]^2 = 0 . \tag{20}
\]

Here \( r = B(x) \) - rigid surface in the cavity form, \( \sigma_{cl}, R_1, B_1 \) - values in the some initial section in the cavity. Estimation on base Eq. (19) show small influence of gas flow on cavity form for ordinary gas injection processes.

2.2 High speed gas flow

Alike to (18) the asymptotic solutions have been obtained on base transonic equation for ideal compressible gas. On this base the system of equations for high speed gas flow in the slender cavity is obtained as:

\[
\left( \frac{2k}{k-1} \right) \left( \frac{P_{cl}}{\rho_{cl}} \right)^{\frac{1}{k}} + U_{cl}^2 = \left( \frac{2k}{k-1} \right) \left( \frac{P_{cl}}{\rho_{cl}} \right)^{\frac{1}{k}} + U_{cl}^2 ,
\]

\[
\left( R^2 - B^2 \right) \rho_{cl} U_{cl} = \left( R_1^2 - B_1^2 \right) \rho_{cl} U_{cl} ,
\]

\[
\rho_{cl} = \rho_{cl} \left( \frac{R}{R_1} \right)^{1/k} ,
\]

where: \( k \) adiabatic curve coefficient, \( R = R_1 \), \( B = B_1 \) cavity and rigid surface radiuses in the initial section, \( \rho_{cl}, P_{cl} \) - gas mass density and pressure in this section. System of equations (20) for given cavity form \( r = R(x) \) finally is transformed to the ordinary equations defining required rigid surfaces in the cavity form \( r = B(x) \). For given rigid surfaces \( r = B(x) \) this system is transformed to the ordinary differential equations nonlinear relay to \( d^2R^2/dx^2 \). System (20) alike to (19) based on the streamline gas flow where speed and pressure are supposed as not changing along radius and are defined by pressure on the cavity. For very high initial speeds \( P_{cl}/U_{cl}^2 \to 0 \) this equation is transformed neglected by small values into the equation:

\[
\frac{d^2R^2}{dx^2} + \frac{\sigma_{cl}}{\mu} \left( \frac{2}{\rho_{cl} U_{cl}^2} \right) \left( \frac{R_1^2 - B_1^2}{R^2 - B^2} \right) = 0 . \tag{21}
\]

Fig. 5 illustrate calculation results on base equation (21) in case of air \( k \approx 1.4 \) for following initial date:

\[
\sigma_{cl} \approx 0.04 , \quad 2P_{cl}/\rho U_{cl}^2 \approx 0.02 , \quad \frac{dR^2}{dx} \bigg|_{\kappa=0} = 0 ,
\]

where for rough estimation it is supposed \( \mu \approx 2 \).

The number of the calculations for the different rigid surface forms and for wide range of Mach Number \( M \) are performed with rough estimation of viscose and another effects. This system is applicable both for air and for very over heat vapor too. As follow from this analysis:

- for high enough supersonic speeds cavity form can be essentially changed under gas flow in cavity influence,
- however thank to gas compressibility we have elastic system where high frequencies oscillations can appear and excite essential wave creation on the cavity surfaces.

3. CAVITATION DRAG

3.1 Cavitator drag

Cavitator drag in general case consists of hydrodynamic and hydrostatic components. The known dependence for the case of a disk-type cavitator is:

\[
D = c_d \pi R_n^2 \frac{\rho U_{\infty}^2}{2} , \quad c_d = c_{d_{th}} + \Delta c_{d_{th}} + c_{d_p} = c_{d_{th}}(1 + \sigma) , \tag{22}
\]
where for a disk, \( c_{d_0} \approx 0.82 - 0.83 \), and the action of the cavity on the hydrodynamic component of drag, \( \Delta c_{dh} \), is not essential.

The unsteady drag component can be roughly estimated based on added mass \( m^*_x \) (\( m^*_x = (4/3)\rho R_b^3 \) for a disk) [2]. The drag of a slender cavitator depends on similar factors more significantly, more details here can be found in the [11].

### 3.2 Cavitation drag

Drag coefficients \( C_D \) and \( C_{Df} (C_{Do}) \) are some characteristics of the supercavitation motion effectiveness. \( C_D \) is drag coefficient relative to the cavity midsection. \( C_{Df} \) for motion in the forward cavity part express drag coefficient of the forward cavity part respect to the section with radius \( R_b \) at \( x = L_0 \), \( \lambda_f = \lambda_n / 2 = L_0 / 2R_b \). Second-order dependencies for like disc cavitators for \( \sigma \to 0 \) (\( \delta / \varepsilon \to 0 \)) are presented below [14, 8, 11].

For \( \sigma \to 0 \), \( C_{Df} = C_{Do} \):

\[
C_{Df} = \frac{c_{d_0} R^2}{R_b^2} = \frac{1}{4(\lambda_n)^2} \ln \left( \frac{4(\lambda_n)^2}{\varepsilon \beta^2} \right) - \frac{1}{16} \ln \left( \frac{\lambda_f^2}{\beta^2} \right) \tag{23}
\]

Drag coefficient for the midsection for \( \sigma \to 0 \) is:

\[
C_D = \frac{c_{d_0} R^2}{R_b^2} = k\sigma = \frac{2\mu}{\lambda^2} \ln \left( \frac{\lambda_f^2}{\beta^2} \right) - \frac{1}{\lambda^2} \ln \left( \frac{\lambda_f^2}{\beta^2} \right) \tag{24}
\]

Taking into account limitations for practically attainable aspect ratios, the drag coefficient, \( C_{Df} \), for forward part of a finite cavity with aspect ratio \( \lambda_f \) is defined as:

\[
C_{Df} = \frac{k\mu}{2} \left[ \frac{1}{\lambda_n^2} \right] \left[ \frac{1}{\lambda_n^2} \ln \left( \frac{\lambda_f^2}{\lambda_n^2} \right) \right] - \left( \frac{2\sigma}{\mu} \right) \frac{\lambda_f^2}{\lambda_n^2} \tag{25}
\]

More perfect as compared to (25) dependence which combine dependencies (23, 24) in case of 2 sectional cavity is:

\[
C_{Df} = \frac{k\mu x}{2(\lambda_n^2)} \left[ \frac{\lambda_f^2}{\lambda_n^2} \ln \left( \frac{\lambda_f^2}{\lambda_n^2} \right) \right] - \left( \sigma - \sigma_1 \right) \tag{26}
\]

Here \( \sigma_1, \sigma_2 \) - cavitation numbers for the first forward part and respectively second back isolated cavity part, \( k, \mu \) defined by (8) correspond \( \sigma_1 = 0.5(\sqrt{\lambda_g} +1/\sqrt{\lambda_g}) \). Dependencies (26) are appropriate for first part of the cavity till up middle section, after middle section the appropriate one is:

\[
C_{Df} = \frac{k\mu x}{2(\lambda_n^2)} \left[ \frac{\lambda_f^2}{\lambda_n^2} \ln \left( \frac{\lambda_f^2}{\lambda_n^2} \right) \right] - \left( \sigma - \sigma_1 \right) \tag{27}
\]

Drag coefficients per unit of volume of the cavity are some effectiveness criteria too. Total drag including cavitator drag can be presented by following expression:

\[
a) \quad D = \frac{\pi c_d R^2}{2}, \quad b) \quad \frac{C_V}{V^{2/3}} = \frac{\pi c_d R^2}{2} \tag{28}
\]

where \( V \) is finite cavity volume which can be defined as:

\[
V = \frac{4\pi k_v}{3\lambda^2} = \frac{4\pi}{3} \left[ 1 + \frac{\ln \left( \varepsilon / \sqrt{\lambda} \right)}{\ln \lambda} \right] \frac{1}{\lambda^2} \tag{29}
\]

For cavitator drag only the volume drag coefficient is defined as (28b), and for forward cavity part, \( \sigma = 0 \) is:

\[
C_{Vo} = \frac{\pi}{16} \left[ \frac{\ln \lambda^2}{\ln \lambda^2} \ln \lambda^2 \right] - \frac{\pi}{16} \left[ \frac{\ln \lambda^2}{\lambda^2} \right] \tag{30}
\]

Volume drag coefficient for finite cavity is defined by following second order dependence:

\[
C_V = \frac{3}{4} \left[ \frac{\ln \lambda^2}{\lambda^2} \right] \ln \lambda^2 - \frac{3}{4} \left[ \frac{\ln \lambda^2}{\lambda^2} \right] \ln \lambda^2 \tag{31}
\]

The averaged near \( \sigma \approx 0.03 \) dependencies are:
motion in a finite cavity taking account of cavitation drag only, the spent mass depends on the spent energy according to the expression (based on Eq. (17)): 

\[ \bar{m}_v = \frac{9 k_o}{4 \pi} \left[ 1 - 2 \pi / \overline{\tau_v} \right] \]

\[ \bar{m}_v = \frac{Q_m}{\rho_u U_{\infty}} , \overline{\tau_v} = \frac{2 DL}{V_p U_{\infty}^2} \]  \hspace{1cm} (35)

Figure 7 illustrates calculation results on applying dependence (35) for two values of velocity, \( U_{\infty} = 10 \text{m/s} \), 50\text{m/s} for \( P_{\infty} \sim 1 \text{atm} \) and a mass density of \( \rho_u \). The cavitation numbers and aspect ratios \( \sigma, \lambda(\sigma) \) are indicated on the graph too. Note the significant decrease of \( m_v \) with \( \tau_v \). As Figure 7 shows for a speed \( \sim 50 \text{m/s} \), \( P_{\infty} \sim 1 \text{atm} \), we have a vapor cavity at \( \sigma \sim 0.08 \).

It would rather the most general here can be the criteria which express specific values of the energy (mass, …) which are required to displace a unit of volume per unit of length (specific energy of the wake): 

\[ e_v = N \frac{U_{\infty}^3}{U_v V} \]

Here \( N \) is power which is spent for motion, \( C_v, C_{vm} \) volume coefficients of spent specific energy and mass. The structure of the general non-dimensional effectiveness criteria can include the terms:

\[ \overline{\tau_v} = e_v / \rho_u V_s^2 / U_b^2 \]  \hspace{1cm} (36)

Here \( k_{oc}, k_{off}, k_1, k_2 \) is defined for concrete hydro-jet propulsion system. Coefficients \( C \) are volume coefficients of spent specific energy and in particular \( C_{vc} \) is the volume drag coefficient which is the same as in dependencies (30 - 33). Structure of the viscous volume drag coefficient here \( C_{vf} \) (\( C_{vof}, C_{vif} \)) is:

\[ C_{vf} = \frac{e_v S}{2 V V_{2/3}} = 2 \chi_{1/3} \lambda \] \hspace{1cm} (38)

where \( S, V \) - typical square and volume which are defined by concrete surface, \( e_v = c_v (Re) \) is friction drag coefficient weakly enough dependent for turbulent flow on Reynolds Number. Here in (37) first term define energy expenses in the out flow marked by index "o" for motion speed \( U_o \) and typical volume \( V_o \), second term is for the inner flow (in hydro-jet propulsion system) typical speed \( U_1 \), volume and gas (fluid) density of the inner flow. Value \( \tau_v \) define thermodynamics loss defined by change of phases, heat transfer and others.
Analogously volume coefficient of gas loss \( C_{vm} \) is defined on base dependence (17): \[ C_{vm} = \frac{k}{2\mu} \left( \frac{3}{4\pi \nu} \right)^{2/3} \left[ \left( 1 - \frac{2\mu}{\lambda^2 E} \right)^{2/3} \right] \] (39) and can be simplified by averaging near \( \sigma \sim 0.03 \): \[ C_{vm} = 0.22 \frac{1 - \sigma / E}{\sigma^{2/3}} \approx 0.09 \left( 1 - \frac{4}{\lambda^2 E} \right)^{2/3} \] (40) More general dependencies can be found apply to motion in the forward cavity part too.

CONCLUSIONS
The supercavitation usage can be effective way of drag reduction for motion in water in case of different types of not large high speed vessels and vehicles. Both application of vapor cavities and cavity with gas injection are possible. Application of usual gas injection way give the possibility to create cavities for less speeds. However for not high speeds and large sizes the influence of the gravity become as essential. The perspective here can be application of high speeds gas injection. The applications of the supercavitation are more effective if more slender cavities are used, but hydrostatic pressure influence can strongly decrease the effectiveness. For cavity creation the energy and mass of gas are spent. This expenses can be reduced with help appropriate closures and changing of the end cavity form.

The drag coefficient per cavity middle section and per unit of volume are effective enough characteristics. However in general case it is difficult to differ how passive or active the different forces can be. So the most effective and general characteristics of the different propulsion systems effectiveness can be values of energy (and gas mass) expenses per unit of the volume for the system transition. This dependencies can give the possibility to estimate effectiveness of different ways of the supercavitation usage apply to drag reduction problems.

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