PLANING FOR THE SUB – AND SUPersonic SPEEDS

Alexander N. MAYBORODA
Professor of Kyiv State-Maritime Academy
ma@ukrpost.net

ABSTRACT
The present paper is devoted to some quantitative estimation of hydrodynamic characteristics of planing plate for high speeds. The plate has finite aspect ratio and moves on compressible fluid with detached and attached nose shock. The applicability of the isentropic equation of a thermodynamic state of water is supposed. The similarity transonic flows of water and perfect gas is shown.

A stream turn angle dependence on a shock sweep, and Mach number, at which shock attaches to a plate, dependence on a trim angle for water is built. Generalized with due regard for free surface of compressible fluid Young's formula for the plate normal force coefficient derivative with respect to hull angle for finite aspect ratio planing plate is obtained.

The numerical results for little hull angles are in keeping with known theoretical and experimental data for an undersurface of a plate and wedge at subsonic, transonic, and supersonic velocity range.

INTRODUCTION
The flow generated by the plate gliding on the incompressible fluid was studied enough [1,2,3 a. o.]. The interest to the flow of the compressible water became apparent recently because of the growing of the body's motion speeds in water [4]. The linearized analysis of sub– and supersonic was performed for the axisymmetric cavity flows [5]. It is desirable to consider the plate gliding on the compressible fluid at subsonic and supersonic speeds with the purpose to stabilize the motion of the high – speed bodies in water.

This paper describes the main theoretical investigation results of the hydrodynamic characteristics and the nose backwater of the high-speed planing plate.

The rectangular plate (Fig. 1) with aspect ratio \( \lambda \geq 0.01 \) glides at the hull angle \( \psi \leq 60^\circ \), sub –, trans - and supersonic speeds with detached and attached nose shock. The Young's method of carrying surface finite aspect ratio effect consideration was generalized for a motion near the free surface of fluid. The numerical results for low hull angles compare reasonably well with known theoretical and experimental results at transonic velocity range [6,7,8].

NOMENCLATURE

\( V_{\infty} \) Flow velocity at infinity
\( p_{\infty} \) Pressure on free surface of fluid
\( \rho \) Mass density
\( M \) Mach number \( M = V_{\infty} / a \)
\( a \) Sound velocity in unperturbed fluid
\( \lambda \) Aspect ratio of plate \( \lambda = l / b \)
\( b \) Plate width
\( l \) Wetted plate length
\( \lambda_0 \) Initial aspect ratio of plate \( \lambda = l_0 / b \)
\( l_0 \) Plate length submerged under unperturbed fluid
\( \psi \) Hull angle
\( C_{s}^* \) Slope of lift curve
\( l_p \) Distance of pressure center from plate back edge
\( v \) Kinematical coefficient of viscosity
\( Re_i \) Reynolds number \( Re_i = V_{\infty} l / \nu \)
\( s \) Entropy
\( \bar{p} \) Local pressure coefficient \( \bar{p} = (p - p_{\infty}) / \rho V_{\infty}^2 / 2 \)

1. INCOMPRESSIBLE FLUID
The planing theory for incompressible fluid is based on analogy between flowed around planing plate and wing undersurface [9]. In linear approach a Wagner analogy establishes, that hydrodynamic loading of planing plate gives a half of loading of corresponding thin wing.

The slope of plate lift curve at little \( \psi \) was found by Sedov [1]

\[
C_{s}^* = \frac{\pi}{1 + 2\lambda}.
\]
A wetted plate length \( l \) because of nose backwater more length \( l_0 \) submerged under unperturbed fluid level was found by Sedov [1]

\[
\frac{l}{l_0} = 0.5(1 + \sqrt{1 + \frac{1}{\lambda_0}}), \quad \lambda_0 = \frac{l_0}{b} \geq 0.01. \tag{2}
\]

Relative distance of pressure center from plate back edge for \( \lambda \geq 0.01 \) and \( \psi \leq 60^\circ \) was found by Wang and Rispin [10]

\[
\bar{l}_D = \frac{l_{in}}{l} = 1 - \frac{0.1415}{\sqrt{\lambda}} - 0.17 \sin(3\psi), \quad \lambda \geq 0.5,
\]

\[
\bar{l}_D = 0.8, \quad \lambda < 0.5. \tag{3}
\]

Pressure resistance coefficient is

\[
C_{x_p} = C_{x_p}^\psi \sin \psi. \tag{4}
\]

Prandtl–Schlichting’s formula gives friction resistance coefficient

\[
C_{s_f} = \frac{0.455}{(\lg Re)^{2.58}}. \tag{5}
\]

2. COMpressible FLUID, \( M \leq 1 \)

At pressures \( p \leq 3.0 \times 10^5 \) Pa \( (M \leq 1.5) \) water equation of state [11] is

\[
p = B(s) \left[ \frac{\rho}{\rho_0} \right]^n - 1, \tag{6}
\]

where \( B = 2.987 \times 10^6 \) Pa in temperature range from 0\(^\circ\) C to 60\(^\circ\) C; \( \rho_0 \) is water density, extrapolated on zero pressure; \( n = 7.15 \).

Prandtl–Glauert's rule for determination of local pressure coefficient \( \overline{p} \) at stagnation point on contour, planing, for example, near \( M = 0.8 \), has an order of mismatch error 50%.

Expression for coefficient \( k_M \), indicating the fractional increase of pressure, caused by Mach effect at subsonic regime, in comparison with incompressible fluid was built for undersurface of plate planing at \( M \leq 1 \)

\[
k_M = \left[ \frac{\overline{p}_{M \leq 1}}{\overline{p}_{M=0}} \right] = \frac{2}{nM^2} \left[ 1 + \frac{n-1}{2} M^2 \right]^{\frac{n}{n-1}} - 1. \tag{7}
\]

Magnitude \( k_M \) for water and air is compared on Fig. 2 with Prandtl-Glauert's coefficient \( \frac{1}{\sqrt{1-M^2}} \), as well as with a relative change of wing undersurface pressure in air [12].

![Fig. 2: Mach effect on profile hydrodynamic loading at subsonic flow](image)

Generalized with due regard for free surface of compressible fluid Young's formula for the plate normal force derivative with respect to hull angle is built for finite aspect ratio plate planing at subsonic speed

\[
C_{x^\psi}^\nu \left|_{M=0} \right. = \frac{\pi k_M C_{x^\psi}^\nu \left|_{M=0} \right.}{\pi + 2\lambda k_M C_{x^\psi}^\nu \left|_{M=0} \right.}, \quad M \leq 1. \tag{8}
\]

For finite hull angles is

\[
C_{x^\psi}^\nu \left|_{M=0} \right. = k_\psi \pi, \tag{9}
\]

where according to nonlinear theory [1] is

\[
k_\psi = \frac{2}{\psi \tan^{-1} \left( \frac{\psi}{2} + \frac{\psi}{2} \ln \left( \tan^{-1} \frac{\psi}{2} \right) \right)}. \tag{10}
\]

An expression for subsonic planing plate relative nose backwater is based on Wagner's immersion theory generalization

\[
\frac{l}{l_0} = \frac{3 + \sqrt{1 + \frac{1}{\lambda_0}}}{nM^2} \left[ 1 + \frac{n-1}{2} M^2 \right]^{\frac{n}{n-1}} - 1, \quad M \leq 1. \tag{11}
\]
The dependence \( \frac{l}{l_0} \) on initial plate aspect ratio \( \lambda_0 \) for diverse numbers \( M \leq 1 \) is represented on Fig. 3.

\[ C_{xf} = \frac{0.455}{(\lg Re)^{0.58}} (1 - 0.07M^{1.5}). \]  

(12)

3. COMPRESSIBLE FLUID, \( M > 1 \)

There are three typical domains of flow along the planing contour at \( M > 1 \):

- \( 1 < M < M' \) – flow with detached nose shock, where \( M' \) – Mach number, near which a maximum turning angle within shock is equal to contour hull angle;

- \( M' \leq M \leq M'' \) – flow with attached curved shock, where \( M'' \) – Mach number, near which a speed from behind shock becomes supersonic;

- \( M'' < M \) – supersonic flow from behind attached shock.

Turning angles \( \alpha \) in shock with shock sweep \( \beta \) at different Mach numbers are showed on Fig. 4.

Hull angles and Mach number \( M' \), near which water and air shock attaches to contour, are showed on Fig. 5. As is obvious a planing contour with hull angles \( \psi > 2.6^0 \) is flowed with detached shock.

\[ \frac{M'}{\psi} \]

(13)

where \( v \) is specific volume and a derivative is undertaken under fixed entropy \( s \), can be taken equal to state's equation index \( n \) for water at \( M \leq 1.5 \). Consequently, the dynamics similarity laws of perfect gas can be used for water with the assumption that gas adiabatic exponent in similarity criterions is replaced by index \( n \). That allows using for water the results of exact decisions and experiments obtained for gas. In particular, regime of attachment of water shock, represented on Fig. 5, comes to an agreement about value Karman-Tsien's similarity criterion obtained for supersonic flow around thin wedge [8].

At \( 1 < M < M' \) the shock is detached and is reflected from free surface by centered Prandtl-Meyer's rarefaction wave. A planing contour is streamlined by subsonic stream from behind shock. Because of absence of supersonic zones on flat contour, it is possible to use the correspondence principle [13] and to build coefficient \( k_{\mu} \) at supersonic regime with detached shock.
An expression for relative nose backwater under supersonic planing with detached shock is obtained on Wagner's immersion theory generalization basis

$$k_M = \frac{2}{nM^2} \left[ \left( \frac{n+1}{2} M^2 \right)^{\frac{n}{n+1}} - 1 \right]$$

$$1 < M < M'.$$ \quad (14)

A relative pressure center on planing contour changes insignificant at $1 < M < M'$. For like flow around thin wedge a displacement of pressure center to back edge forms 4% contour chord [7].

A shock attaches to contour in its intersection with unperturbed free surface of fluid at $M = M'$. Flow from behind shock is subsonic at $M = M'$ and later becomes supersonic at $M = M''$. A magnitude of space $\Delta M = M'' - M'$ depends on Mach number and forms at $M \leq 1.5$ for water at most 0.05. Therefore, one can be used to practical application as number $M_A$ when a shock attaches and a speed from behind shock becomes supersonic a middle magnitude

$$M_A = 0.5(M' - M'').$$ \quad (16)

Then for computation of the plate lift curve slope at $M \leq 1.5$ with attached shock well known expression can be used

$$C_{\psi, \beta} = \frac{4}{\psi(n+1)} \left( \frac{\sin^2 \beta - \frac{1}{M^2}}{\psi^2} \right),$$

$$M_A \leq M \leq 1.5,$$ \quad (17)

where a shock sweep $\beta$ determines by iteration method from equation

$$\tan \psi = \frac{2 \left( (M^2 - 1) \tan^2 \beta - 1 \right)}{\left( nM^2 + 2 \right) \left( \frac{n+1}{2} M^2 + 2 \tan \beta \right)}.$$ \quad (18)

On Fig. 6 with transonic similarity criterions

$$\bar{M} = \left( \frac{n+1}{\psi^2} \right)^{\frac{1}{1-\frac{n}{n+1}}} \left( \frac{M}{n+1} \right)^{\frac{1}{2-\frac{n}{n+1}}}$$

and

$$\bar{C}_\psi = (n+1)^{\frac{1}{2}} \psi \bar{C}_\psi$$

the by a suggested program computing data of planing at $0.95 \leq M \leq 1.25$ with little hull angles the contour loading are compared to results Guderley [6], Vincenti and Wagoner [7] and Yoshihara [8].

In accepted designations the motions at $M'$ and $M''$ conform to criterion value $\bar{M}' = 1.18$ and $\bar{M}'' = 1.26$. The motion at $M = 1$ conforms to value $\bar{M} = 0$. Vincenti and Wagoner have researched a flow around thin wedge at $0 < \bar{M} < 1.05$ with detached shock and experimentally confirmed their results. Yoshihara has discussed the flow around a thin wedge at $1.18 \leq \bar{M} \leq 1.26$ (from the shock attaching when a speed behind the shock is subsonic to the oblique shock formation with a supersonic speed on the wedge). At $1.05 < \bar{M} < 1.18$ when the shock thickness is vanishing, the interpolated values are demonstrated in Fig. 6. The results of the above mentioned researches are represented in Fig. 6 according to the linear theory for a half of the wedge loading. The values $\bar{C}_\psi$ for the supersonic flow with the attached shock at $\bar{M} > 1.26$ shown in Fig. 6 practically coincide with the results obtained by Guderley [13]. The Guderley's result [6] for the flow around to plate undersurface at $M = 1$ is demonstrated in Fig. 6 by a point.

A stagnation point and nose backwater on the planing contour vanishes at the supersonic flow behind the attached shock. A pressure center moves to the middle of the wetted length.
A relative nose backwater for the plate planing at initial aspect ratio \( \lambda_0 = 1 \) and hull angle \( \psi = 2^\circ \) is showed in Fig. 7 depending on Mach number \( M \). A value \( M = 1.38 \) in Fig. 7 conforms to shock attaching.

The effect of the finite aspect ratio of the plate was realized by the Young's generalized formula for a flow with the detached shock and by the Hilton's method [12] for the flow with attached shock:

with detached shock \( 1 < M < M_\Delta \)

\[
C_n^* \big|_{M<1} = \frac{\pi k_M}{\pi + 2\lambda k_M} C_n^{\psi=0} \big|_{M<1};
\]  \hspace{1cm} (19)

with attached shock \( M_\Delta \leq M \leq 1.5 \)

\[
C_n^* \big|_{M=M_\Delta} = \frac{4}{\psi(n+1)} \left( \sin^2 \beta - \frac{1}{M^2} \right) \left[ 1 - \frac{1}{2R} \right],
\]

\[
R = \lambda \sqrt{M^2 - 1} > 1; \hspace{1cm} (20)
\]

\[
C_n^* \big|_{M=M_\Delta} = \frac{4}{\psi(n+1)R} \left( \sin^2 \beta - \frac{1}{M^2} \right) \left[ (2R - 1) \arcsin R + R(R - 2) \ln \frac{1 + \sqrt{1 - R^2}}{R} + (1 + R) \sqrt{1 - R^2} \right]
\]

\[
0.5 < R \leq 1. \hspace{1cm} (21)
\]

The hydrodynamic coefficients of the plate planing at \( 0 \leq M \leq 1.5 \) with the various initial aspect ratio and hull angles are represented in Fig. 8 and 9. The conditions with the detached shock are showed in Fig. 8. The conditions with the attached shock are showed in Fig. 9.

REFERENCES


